IMAGE CONVERSION OF SIGMA- AND PI-COMPONENTS OF RADIATION OF RELATIVISTIC ELECTRONS AND METROLOGICAL CHARACTERISTICS OF THE PHOTON FLUX IN THE SYNCHROTRON RADIATION OUTLET

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Synchrotron radiation (SR), which occurs during the operation of electron storage rings, makes it possible to carry out fundamental scientific research and applied technological work, has unique properties: a continuous spectrum from infrared to X-ray regions, acute directivity, a high degree of polarization, the ability to accurately calculate characteristics [1-3]. Due to the unique properties listed above, SR has recently begun to play a decisive role in the intensive development of the most promising scientific research in physics, chemistry, biology, medicine, microelectronics, tomography, materials science, etc. [4-6]. SR from bending magnets with precisely defined parameters can act as an absolute standard in metrology [7].

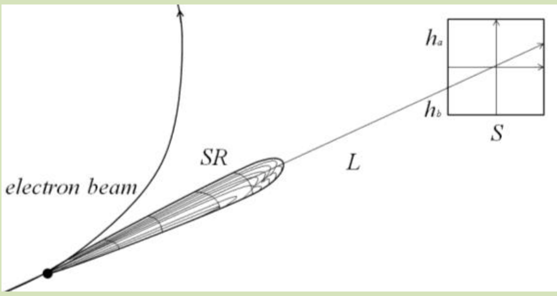

Fig. 1. Scheme for observing the flux of SR quanta

In [8], the influence of the distribution of particles along the vertical and vertical oscillations in the beam on the formation of the angular distribution of SR was investigated. In this work, the efficiency of capture of the flux of σ- and π-components by an optical window in the channel for extracting SR (see Fig. 1) is studied and the dependence of the capture quality for different radiation wavelengths is analyzed. Examples of numerical calculations for the formation of the final SR spectral density of a relativistic electron with an energy of 225 MeV at the output of the optical channel in generator "NESTOR" are presented [9-11].

MATHEMATICAL MODEL

The SR of a relativistic electron is characterized by a high degree of polarization. In particular, at zero angle $(\psi = 0)$ to the orbital plane, it is linearly polarized. The spectral-angular dependences of the flux of SR quanta are calculated in accordance with the expressions that describe the distribution density $w_\sigma(\psi)$ (in the plane of the orbit) and the π-component of the radiation $w_\pi(\psi)$ (perpendicular to the plane) [2]:

$$w_\sigma(\psi) = G(1+\gamma^2\psi^2)^2 K_{2/3}^2\left(\frac{\lambda_c}{2\lambda}(1+\gamma^2\psi^2)^{3/2}\right),$$
$$w_\pi(\psi) = G\gamma^2\psi^2(1+\gamma^2\psi^2)\, K_{1/3}^2\left(\frac{\lambda_c}{2\lambda}(1+\gamma^2\psi^2)^{3/2}\right), \tag{1}$$

where $= 8\pi e_0^2 R^2 f/3c\hbar\lambda^3\gamma^4$, $K_{2/3}(.)$ and $K_{1/3}(.)$ are the Macdonald functions, λ is the wavelength, E is the electron energy, E_0 is the electron rest energy, $\gamma = E/E_0$ is the relativistic factor, e_0 is the electron charge, R is the radius of rotation of the magnets, f is the frequency of revolution, c is the speed of light, $\hbar$ is the Planck constant, $\lambda_c = 4\pi e_0^2 Rf/(\sqrt{3}c\hbar\gamma^3)$ is the critical wavelength of radiation. The total angular density is $w(\psi) = w_\sigma(\psi) + w_\pi(\psi)$.

The photon flux of each of the electrons is characterized by an angular distribution, the axis of which coincides with the direction of motion of the particle, and the top of the distribution coincides with the place of radiation. In Fig. 2 shows a family of angular distributions of flux density for the σ- and π-components of polarization, calculated for one of the SR output channels in the generator. The calculation parameters in this figure and the following ones were E=225 MeV, R=0.5 m and f=19.46 MHz, while $\lambda_c = 2.45\times10^{-8}$ m.

The electrons in the storage ring vibrate around the equilibrium orbit. These oscillations are due to the recoil of electrons during the emission of SR quanta, as well as intrabeam scattering and scattering by residual gas particles. As a result, the beam particles are distributed around an equilibrium orbit with a normal law in 6-dimensional space [2].

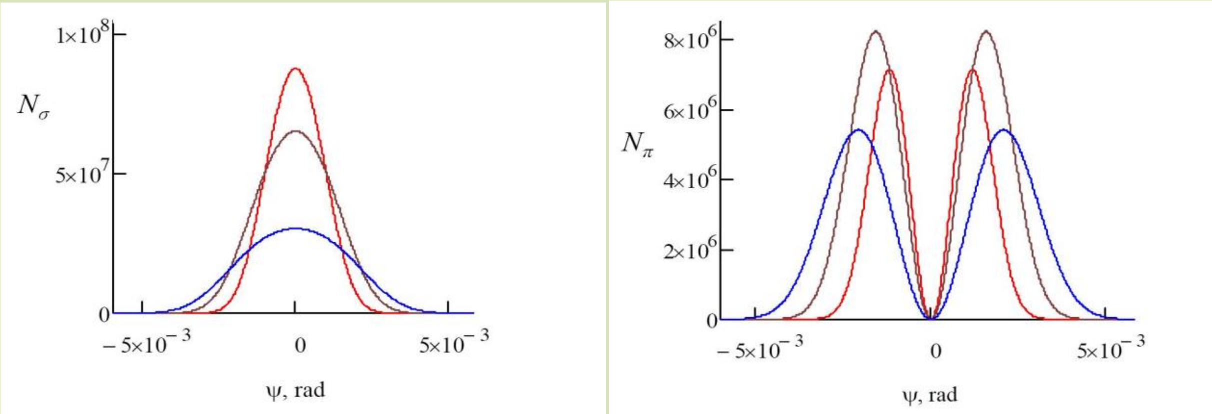

Fig. 2. Angular distributions of the flux of quanta $N_\sigma(\psi)$ and $N_\pi(\psi)$ for different wavelengths; $\lambda = 0.5\,\lambda_c$ *(red),* $\lambda = \lambda_c$ *(brown),* $\lambda = 2\,\lambda_c$ *(blue)*

Let us consider the influence of the distribution of particles in the beam on the properties of the flux of SR quanta. The distribution in the longitudinal direction does not affect the spectral-angular characteristics of the flux of SR quanta due to the azimuthal symmetry. For the same reason, the radial distribution of particles also does not affect the characteristics of the SR flux. For the vertical distribution of particles, we use the formula [2]:

$$\rho(y, y') = \frac{1}{2\pi\sigma_y\sigma_{y'}} \exp\left(-\frac{y^2}{2\sigma_y^2} - \frac{y'^2}{2\sigma_{y'}^2}\right), \tag{2}$$

where σ_y and $\sigma_{y'}$ are the root-mean-square dimensions of the beam along y and y' respectively.

Consider at the base distance L the receiving plane perpendicular to the tangent to the radiation point in a circular orbit. We place at a distance L an optical window with a width S and a vertical aperture (h_a, h_b) along the vertical.

The angle of emission of a quantum ψ, as well as the coordinates of emission $(y,\ y')$ and reception h in the vertical direction, are related by the relation $h - \psi L = y + Ly'$. On average, the total number N_σ and N_π of photons of the σ- and π-polarization components passing through the window will be:

$$N_\sigma = \int_S ds \int_{h_b}^{h_a} dh \int_{-\pi/2}^{\pi/2} d\psi\ \langle \delta(h - y - y' - \psi L)\rangle\, w_\sigma(\psi),$$
$$N_\pi = \int_S ds \int_{h_b}^{h_a} dh \int_{-\pi/2}^{\pi/2} d\psi\ \langle \delta(h - y - y'L - \psi L)\rangle\, w_\pi(\psi), \tag{3}$$

where $\delta(.)$ is the Dirac delta-function. In expression (3), angle brackets denote averaging over y and y' according to (2). Taking into account the width of the receiving window leads to a multiplier:

$$\mu = \frac{S}{2\pi\sqrt{R^2 + L^2}}, \tag{4}$$

so:

$$N_\sigma = \mu \int_{h_b}^{h_a} dh \int_{-\pi/2}^{\pi/2} d\psi\ \langle \delta(h - y - y'L - \psi L)\rangle\, w_\sigma(\psi),$$
$$N_\pi = \mu \int_{h_b}^{h_a} dh \int_{-\pi/2}^{\pi/2} d\psi\ \langle \delta(h - y - y'L - \psi L)\rangle\, w_\pi(\psi). \tag{5}$$

Bearing in mind (2), we note that for a fixed angle ψ due to the normality y and y' the random variable h is also normal with the mathematical expectation ψL and dispersion:

$$\sigma_L^2 = \sigma_y^2 + \sigma_{y'}^2 L^2. \tag{6}$$

Therefore, for the angular distributions averaged relative to the beam, we obtain:

$$N_\sigma(h) = \frac{\mu L}{\sqrt{2\pi}\sigma_L} \int_{-\pi/2}^{\pi/2} d\psi \exp\left(-\frac{(\psi L - h)^2}{2\sigma_L^2}\right) w_\sigma(\psi),$$
$$N_\pi(h) = \frac{\mu L}{\sqrt{2\pi}\sigma_L} \int_{-\pi/2}^{\pi/2} d\psi \exp\left(-\frac{(\psi L - h)^2}{2\sigma_L^2}\right) w_\pi(\psi). \tag{7}$$

It follows from (7) that the resulting angular distribution is the convolution of the normal density associated with the beam with an angular distribution that describes the emission of quanta of the σ- and π-components of the SR.

Due to the parity, the first moment of distributions (7) is equal to zero. The variance of the resulting angular distribution $\langle h^2/L^2\rangle$ will decrease with increasing L as $\sigma'^2_L = \sigma^2_{y'} + \sigma^2_y/L^2$.

The distribution of quanta for the σ-component of polarization, due to its unimodality, is more resistant to such an influence. Since the angular spectrum of the π-component of polarization has two symmetric maxima, therefore, its broadening by virtue of (7) turns out to be more noticeable. In this case, with an increase in the base distance L, the spatial picture along the vertical axis will also expand. As the wavelength λ decreases, there is a corresponding decrease in the angular dispersion of the π-component $\langle\psi^2_\pi(\lambda)\rangle$, therefore, this polarization component will be normalized if $\sigma^2_{y'} >> \langle\psi^2_\pi(\lambda)\rangle$. Otherwise, the angular distribution will not normalize for any L.

Let us perform the integration over the vertical coordinate h in the formula (7) within the limits $(h_a,\ h_b)$, using the integration variable $t = (h - \psi L)/\sqrt{2}\sigma_L$. Then, for the total number of photons N_σ and N_π of polarization components, we find:

$$N_\sigma = \frac{\mu}{2}\int_{-\pi/2}^{\pi/2} d\psi\ \mathrm{E}_1(\psi, h_a, h_b, L)\, w_\sigma(\psi),$$
$$N_\pi = \frac{\mu}{2}\int_{-\pi/2}^{\pi/2} d\psi\ \mathrm{E}_1(\psi, h_a, h_b, L)\, w_\pi(\psi), \tag{8}$$

where the function is introduced:

$$\mathrm{E}_1(\psi, h_a, h_b, L) = \mathrm{erf}\left(\frac{\psi L - h_a}{\sqrt{2}\sigma_L}\right) - \mathrm{erf}\left(\frac{\psi L - h_b}{\sqrt{2}\sigma_L}\right) \tag{9}$$

and erf(.) is the error function.

In the symmetric case, when $h_a = -h_b = H$, expressions (8) are simplified:

$$N_\sigma = \mu\int_{-\pi/2}^{\pi/2} d\psi\ \mathrm{erf}\left(\frac{\psi L - H}{\sqrt{2}\sigma_L}\right)\, w_\sigma(\psi),$$
$$N_\pi = \mu\int_{-\frac{\pi}{2}}^{\pi/2} d\psi\, \mathrm{erf}\left(\frac{\psi L - H}{\sqrt{2}\sigma_L}\right)\, w_\pi(\psi). \tag{10}$$

Expressions (8)-(10), due to the rather small value of the variance σ^2_L are difficult to use in numerical calculations. Therefore, we will bring expressions (8) to a more convenient and visual form. Namely, we use the integral functions of the angular distribution of the components. In Fig. 3 shows a family of integral angular distributions of flux density σ- and π-components of polarization calculated for one of the SR output channels in the generator.

$$W_\sigma(\psi) = \int_{-\pi/2}^{\psi} d\psi'\ w_\sigma(\psi'),$$
$$W_\pi(\psi) = \int_{-\pi/2}^{\psi} d\psi'\ w_\pi(\psi'). \tag{11}$$

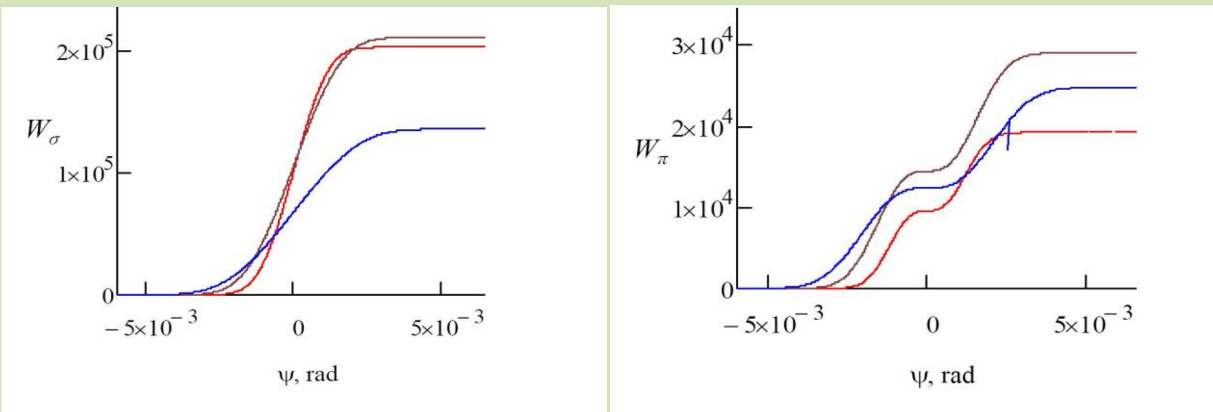

Fig. 3. Integral angular distributions of the flux of quanta $W_\sigma(\psi)$ and $W_\pi(\psi)$; $\lambda = 0.5\ \lambda_c$ (red),$\lambda = \lambda_c$ (brown), $\lambda = 2\ \lambda_c$ (blue)

The calculation parameters in Fig. 3 and following were σ_y =0.2 mm and $\sigma_{y'}$ =0.15 mrad.

Using functions (11), we take the integrals in (8) by parts. As a result, we get:

$$N_\sigma = \frac{\mu}{2}\int_{-\pi/2}^{\pi/2} d\psi\ \mathrm{E}_2(\psi, h_a, h_b, L)\, W_\sigma(\psi),$$

$$N_{\pi} = \frac{\mu}{2}\int_{-\pi/2}^{\pi/2} d\psi \mathrm{E}_2(\psi, h_a, h_b, L)\, W_{\pi}(\psi) \tag{12}$$

where the function is introduced:

$$\mathrm{E}_2(\psi, h_a, h_b, L) = \frac{1}{\sqrt{2\pi}\sigma_L} \times$$
$$\times \left[\exp\left(-\frac{(\psi L - h_a)^2}{2\sigma_L^2}\right) - \exp\left(-\frac{(\psi L - h_b)^2}{2\sigma_L^2}\right)\right]. \tag{13}$$

Due to the smallness of σ_L the exponents in (13) have the filtering property. Therefore, only the regions around the points $\psi_a = h_a/L$ and $\psi_b = h_b/L$ of size $(-3\sigma_L, 3\sigma_L)$ each. Then integrals (12) take the form:

$$N_\sigma = \frac{\mu}{\sqrt{2\pi}\sigma_L} \times$$
$$\int_{-3\sigma_L}^{3\sigma_L} d\xi \exp\left(-\frac{\xi^2}{2\sigma_L^2}\right)\left[W_\sigma\left(\frac{h_a+\xi}{L}\right) - W_\sigma\left(\frac{h_b+\xi}{L}\right)\right],$$
$$N_\pi = \frac{\mu}{\sqrt{2\pi}\sigma_L} \times \tag{14}$$
$$\int_{-3\sigma_L}^{3\sigma_L} d\xi \exp\left(-\frac{\xi^2}{2\sigma_L^2}\right)\left[W_\pi\left(\frac{h_a+\xi}{L}\right) - W_\pi\left(\frac{h_b+\xi}{L}\right)\right].$$

It follows from (14) that for $\sigma_L \to 0$ the leading terms are equal to:

$$N_\sigma = \mu[W_\sigma(\psi_a) - W_\sigma(\psi_b)],$$
$$N_\pi = \mu[W_\pi(\psi_a) - W_\pi(\psi_b)] \tag{15}$$

and correspond to the boundary angles ψ_a and ψ_b of capture of the flux of quanta by the window. Due to the parity with respect to of ψ the angular functions $w_\sigma(\psi)$ and $w_\pi(\psi)$ the terms linear in σ_L vanish.

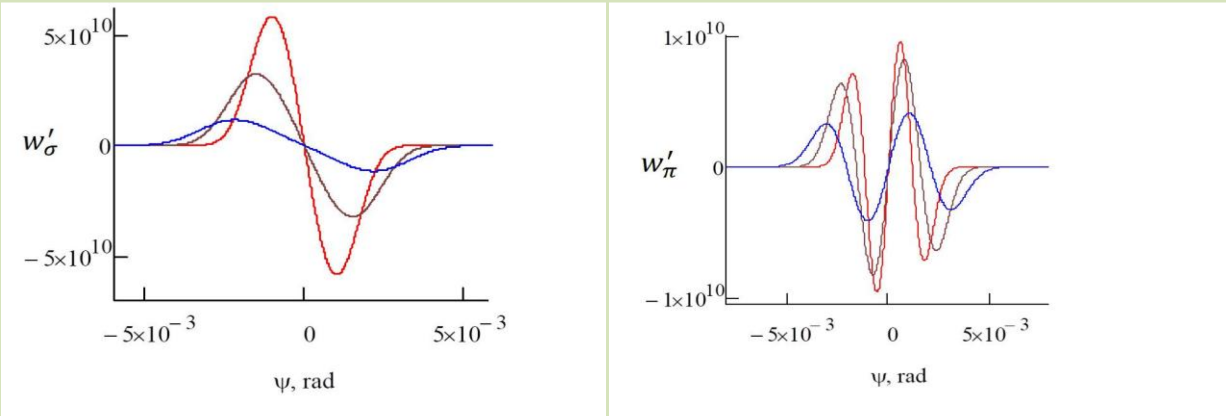

Fig. 4. Derivatives of distributions $w'_\sigma(\psi)$ and $w'_\pi(\psi)$; $\lambda = 0.5\,\lambda_c$ (red), $\lambda = \lambda_c$ (brown), $\lambda = 2\,\lambda_c$ (blue)

Corrections quadratic in σ_L are:

$$\Delta^{(2)}N_\sigma = \mu\frac{\sigma_L^2}{2L^2}[w'_\sigma(\psi_a) - w'_\sigma(\psi_b)],$$
$$\Delta^{(2)}N_\pi = \mu\frac{\sigma_L^2}{2L^2}[w'_\pi(\psi_a) - w'_\pi(\psi_b)], \tag{16}$$

where $w'_\sigma(\psi)$ и $w'_\pi(\psi)$ are the derivatives of the angular functions $w_\sigma(\psi)$ and $w_\pi(\psi)$ (see Fig. 4).

In the symmetric case, when $h_a = -h_b = H$, expressions (10)-(12) admit the representation:

$$N_\sigma = \mu\left(W_\sigma(\psi_a) - W_\sigma(-\psi_a) + \frac{\sigma_L^2}{L^2} w'_\sigma(\psi_b)\right),$$
$$N_\pi = \mu\left(W_\pi(\psi_a) - W_\pi(-\psi_a) + \frac{\sigma_L^2}{L^2} w'_\pi(\psi_b)\right). \quad (17)$$

NUMERICAL RESULTS

Let us present the results of a numerical calculation of the spectral-angular flux of the number of emitted SR quanta. The calculations were carried out for the case of a window with a width *S*=0.06 m and vertical dimensions *H*=0.03 m. Thus, at the base distance *L*=3 m, the capture angle was ψ_a=10 mrad.

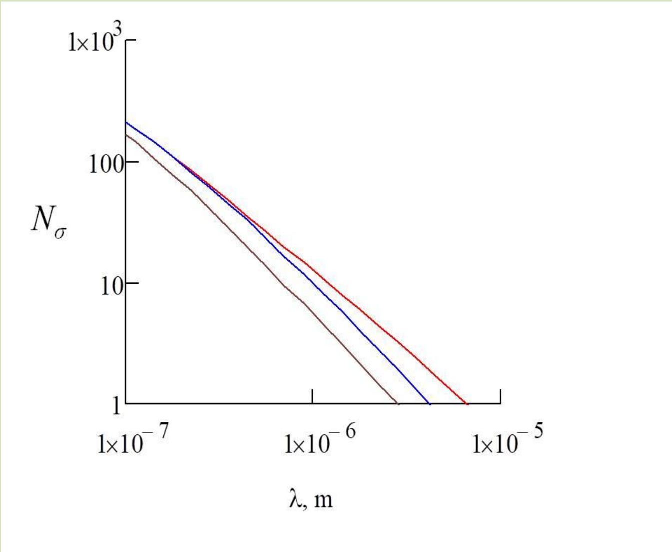

Fig. 5. Flux of quanta of σ-component N_σ at H=0.05 m (red), H=0.02 m (brown), H=0.01 m (blue)

The calculations were carried out under the assumption that 1 electron revolves in the orbit. In Fig. 5 shows the dependence of the number N_σ of captured quanta on the wavelength λ of radiation for different capture angles. For comparison, the dependences are shown for the cases *H*=0.05 m, *H*=0.02 m and *H*=0.01 m. It can be seen that starting from the wavelength $\lambda=1\times10^{-7}$ m, the number of photons decreases, falling at $\lambda=1\times10^{-5}$ m by about 100 times.

Similar spectral dependences for the number N_π captured quanta on the wavelength λ are shown in fig. 6.

In the case of the π-component, the decrease is more noticeable; moreover, it begins for shorter wavelengths $\lambda=10^{-8.5}$ m. This is due to the fact that the peripheral parts of the angular distribution $w_\sigma(\psi)$ are more pronounced than that of the distribution $w_\pi(\psi)$.

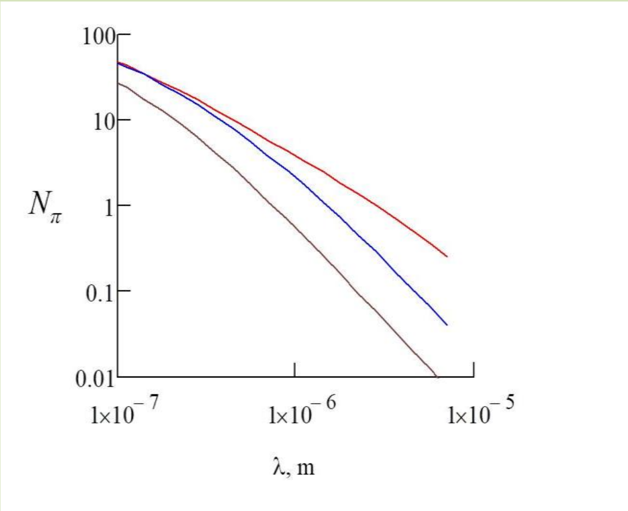

Fig. 6. Flux of quanta of π-component N_π, at H=0.05 m (red), H=0.02 m (blue), H=0.01 m (brown)

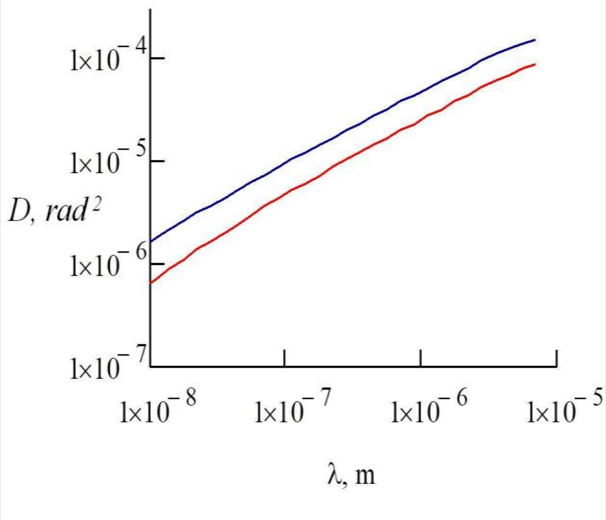

Fig. 7. Dispersions D of angular distributions of the σ-component $D_\sigma(\lambda)$ (red) и π-компоненты $D_\pi(\lambda)$ (blue)

From Fig. 2 it can be seen that the density of the π-component occupies a larger angular interval than the density of the σ- component. The calculated dependences for the variances of the angular distributions $D_\sigma(\lambda) = \langle\psi_\sigma^2(\lambda)\rangle$ и $D_\pi(\lambda) = \langle\psi_\pi^2(\lambda)\rangle$ are shown in Fig. 7

From Fig. 7 that the angular dispersion of the-component is up to two times the angular dispersion of the σ-component. This property leads to less efficient capture of π-component photons into the window.

CONCLUSION

From the given dependences it follows that the main factors determining the capture of the flux of quanta in the channel are the vertical size H of the window and the standard deviations of the angular (SMD) distributions $S_\sigma(\lambda) = D_\sigma^{1/2}(\lambda)$ and $S_\pi(\lambda) = D_\pi^{1/2}(\lambda)$. With an increase in the wavelength λ the values $LS_\sigma(\lambda)$ and $LS_\pi(\lambda)$ approach the value of H and the capture efficiency is also influenced by the peripheral parts of the distributions $w_\sigma(\psi)$ and $w_\pi(\psi)$.

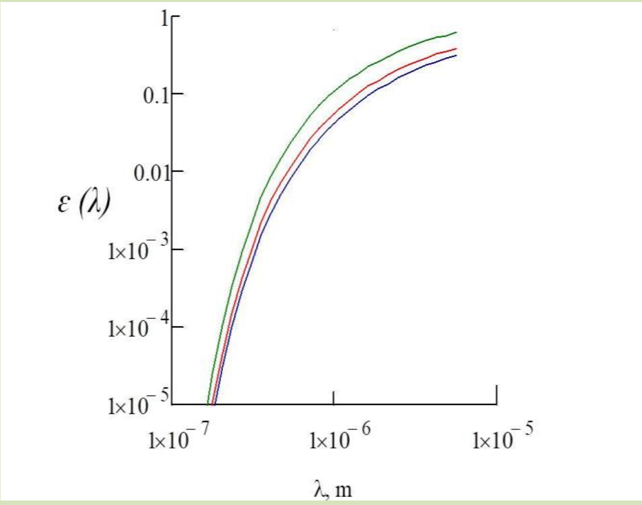

Fig. 8. Fraction $\varepsilon(\lambda)$ of uncaptured quanta N_σ (blue), N_π (green) and total flux of quanta $N_\sigma + N_\pi$ (red)

In Fig. 8 shows the dependences ε(λ) of the fraction of quanta that are not captured in the window on the wavelength λ. It can be seen that for wavelengths $\lambda \leq 5\times10^{-7}$ m capture into a window with an angular opening (-3; 3) mrad takes place with a sufficiently high efficiency. With a further increase in the wavelength, the fraction of uncaptured quanta increases. Moreover, this value is larger for quanta of the π-component, which is explained by the fact that the standard deviation of the π-componentalways exceeds the standard deviation $S_\pi(\lambda)$ always exceeds the standard deviation $S_\sigma(\lambda)$ σ-component of the SR.

In channels with an increased length L, the decisive influence on the formation of angular distributions in the receiving plane and, accordingly, the efficiency of the capture of photon fluxes will be exerted by the standard deviation of the angular distributions $S_\sigma(\lambda)$ and $S_\pi(\lambda)$.