POLARIZATION OF DIFFRACTION RADIATION FROM THE CHARGED PARTICLE AND THE BUNCH OF PARTICLES ON THE METAL SPHERE

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INTRODUCTION

Diffraction radiation (DR) arises when a charged particle moves uniformly near the inhomogeneity of the electromagnetic properties of the medium, the simplest case is the motion of a particle near a conducting half-plane in vacuum [1]. This phenomenon can be used for non-destructive diagnostics of charged particle beams [2]. The approach to description of DR of a nonrelativistic charged particle on an ideally conducting sphere based on the method of images known from electrostatics had been proposed in [3]. In [4, 5], this approach was used to find the polarization of the radiation generated by a separate particle passing by the sphere. In this work, we consider the polarization characteristics of coherent diffraction radiation generated on a sphere by a pancake-bunch of nonrelativistic charged particles and show how measuring the radiation polarization makes it possible to diagnose the position of the bunch edges. Note that the diagnostics of bunches of nonrelativistic particles using the long-wavelength transition and diffraction radiation generated by them on various targets is widely discussed in the current literature (e. g. [6] and references therein).

DIFFRACTION RADIATION ON A SPHERE AND ITS POLARIZATION

The method of image charges consists in reproducing the field created by a charged particle in the presence of conducting bodies by adding one or more imaginary charges. In the case of a rectilinear and uniform motion of a real charge near a conducting sphere, its image will move with acceleration Fig. 1) that generates the radiation.

The amplitude of the vector potential of the radiated wave is proportional to (see e. g. [7])

$$\mathbf{I} = \int_{-\infty}^{\infty} e(t)\,\mathbf{v}(t)\,e^{i(\omega t - \mathbf{kr}(t))}\,dt, \tag{1}$$

where ω and $\mathbf{k}$ are the frequency and the wave vector of the radiation, $|\mathbf{k}| = \omega/c$, $e(t)$, $\mathbf{r}(t)$ and $\mathbf{v}(t)$ are the value, position and speed of the image charge. The spectral-angular density of the radiation with a certain polarization is described by the formula

$$\left(\frac{d\mathcal{E}}{d\omega d\Omega}\right)_\alpha = \frac{\omega^2}{4\pi^2 c^3}\left|\mathbf{e}_\alpha \cdot \mathbf{I}\right|^2, \tag{2}$$

where $\mathbf{e}_\alpha$ ($\alpha = 1,\ 2$) are the polarization vectors that orthogonal to $\mathbf{k}$ and to each other:

$$\mathbf{e}_1 = \frac{\mathbf{k}\times\mathbf{v}_0}{|\mathbf{k}\times\mathbf{v}_0|} = \mathbf{e}_x\sin\varphi - \mathbf{e}_y\cos\varphi\,, \quad \mathbf{e}_2 = \frac{\mathbf{k}\times\mathbf{e}_1}{k} = \mathbf{e}_x\cos\theta\cos\varphi + \mathbf{e}_y\cos\theta\cos\varphi - \mathbf{e}_z\sin\theta, \tag{3}$$

where $\mathbf{v}_0$ is the velocity of the incident particle.

The maximum radiation intensity of the nonrelativistic incident particle is located in large wavelengths domain,

$$\lambda \gg 2\pi R^2/b\,, \qquad \omega \ll cb/R^2\,, \tag{4}$$

where R is the radius of the sphere, $b = \sqrt{x^2+y^2}$ is the impact parameter of the incident particle. The components of $\mathbf{I}$ in this case are equal [4]

$$I_x^{(1)} = \frac{4}{3}e_0R^3\frac{\omega^2}{v_0^2}\,i\,\frac{x}{b}K_1\left(\frac{\omega}{v_0}b\right), \quad I_y^{(1)} = \frac{4}{3}e_0R^3\frac{\omega^2}{v_0^2}\,i\,\frac{y}{b}K_1\left(\frac{\omega}{v_0}b\right), \quad I_z^{(1)} = -\frac{4}{3}e_0R^3\frac{\omega^2}{v_0^2}\left[K_0\left(\frac{\omega}{v_0}b\right) + \frac{v_0}{2\omega b}K_1\left(\frac{\omega}{v_0}b\right)\right]. \tag{5}$$

where e_0 is the charge of the incident particle, $K_0(x)$ and $K_1(x)$ are the modified Bessel functions of the second kind (McDonald functions). The dimensionless quantity $(4\pi^2c/e_0^2)\sum_\alpha(d\mathcal{E}/d\omega d\Omega)_\alpha$ that characterizes the spectral-angular radiation density summed over the polarizations is presented in the figure 2 in the form of directional diagram. The maximum spectral radiation density is achieved with a combination of parameter values $\omega b/v_0 \approx 2.34$ [3].

The polarization tensor [8] will be equal to

$$\rho_{\alpha\beta} = (\mathbf{e}_\alpha\cdot\mathbf{I})\,(\mathbf{e}_\beta\cdot\mathbf{I})^*/\,\mathrm{Sp}(\mathbf{e}_\alpha\cdot\mathbf{I})\,(\mathbf{e}_\alpha\cdot\mathbf{I})^*, \tag{6}$$

and the Stokes parameters ξ_1, ξ_2, ξ_3 characterizing the radiation polarization are

$$\rho_{\alpha\beta} = \frac{1}{2}\begin{pmatrix}1+\xi_3 & \xi_1 - i\xi_2\\ \xi_1 - i\xi_2 & 1-\xi_3\end{pmatrix}, \tag{7}$$

where $\xi_1 = l\sin 2\phi$, $\xi_2 = A$, $\xi_3 = l\cos 2\phi$, l is the degree of maximum linear polarization, ϕ is the angle between the direction of maximum linear polarization and the vector $\mathbf{e}_1$, A is the degree of circular polarization.

In the case of a single incident particle $\xi_1^2+\xi_2^2+\xi_3^2 = 1$, so the radiation is completely polarized (elliptically in general), and linearly in the plane containing the center of the sphere and the trajectory of the incident particle (blue plane in the fig. 1). On the other hand, for radiation directions perpendicular to this plane and close to them (within a very wide range), the radiation polarization will be close to circular (right for $y < 0$ and left for $y > 0$, depending on the sign of the value ξ_2). That creates a possibility for monitoring the trajectory of a passing particle [4].

Let the center of the sphere and the detector sensitive to the radiation polarization lie in the plane (x, z) (fig. 3). Then, if the trajectory of the particle lies in the same plane (x, z), the detector will register linear polarization (fig. 3, left). If the trajectory of a particle passes parallel to the plane (x, z) to the right or to the left (fig. 3, right), an admixture of circular polarization of the corresponding sign depending on the azimuth of the trajectory will appear in the recorded radiation. Thus, the registration of the polarization of the diffraction radiation on the sphere will make it possible to determine the ratio between the x and y components of the two-dimensional impact parameter of the particle, while the total radiation intensity in a given direction will be determined by the absolute value of the impact parameter $b = \sqrt{x^2+y^2}$.

POLARIZATION OF COHERENT DIFFRACTION RADIATION OF A BUNCH

Consider now the coherent radiation of a pancake-bunch of charged particles with the charge density distribution function $Nn(x, y)$, where N is the total number of particles in the bunch, and $n(x, y)$ is normalized function:

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} n(x,y)\,dxdy = 1\,.$$

In the case when the size of the bunch is much less than the radiation wavelength, the components of $\mathbf{I}$ describe the coherent radiation of the bunch and can be found by integrating (5) with the distribution function:

$$I_i = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} I_i^{(1)}(x,y)n(x,y)\,dxdy\,.$$

In particular, for the component I_y we obtain after integration by parts

$$I_y = N\frac{4}{3}e_0R^3\frac{\omega^2}{v_0^2}\,i\,\frac{v_0}{\omega}\int K_0\left(\frac{\omega}{v_0}\sqrt{x^2+y^2}\right)\frac{\partial n(x,y)}{\partial y}\,dxdy\,. \tag{8}$$

Thus the radiation polarized in the direction of the y-axis will be maximum when the maximum change in the density of the bunch of particles in the direction of the y-axis (that is left or right boundary of the bunch) passes over the top point of the sphere. This creates the possibility of detecting the edges of a bunch.

In the case when the radiation detector is located above the top point of the sphere ($\theta = \pi/2$, $\varphi = 0$) unit polarization vectors are equal $\mathbf{e}_1 = -\mathbf{e}_y$, $\mathbf{e}_2 = -\mathbf{e}_z$. Figures 4 and 5 show the characteristics of the radiation in this direction for a homogeneous rectangular bunch and an elliptical bunch, respectively. It can be seen that the maximum radiation polarized in the direction of the y-axis will be observed when the right or left edge of the bunch passes over the top point of the sphere, as expected from (8). Therefore, by moving the sphere and the detector and registering the intensity of radiation polarized in the direction of the y-axis, we can find in a non-destructive way the positions of the left and right edges of the bunch, and measure its size in the direction of the y-axis. In addition, the appearance of a significant degree of circular polarization of radiation or a significant decrease in the magnitude of linear polarization can serve as evidence of the proximity of the edge of the bunch to the upper point of the sphere.

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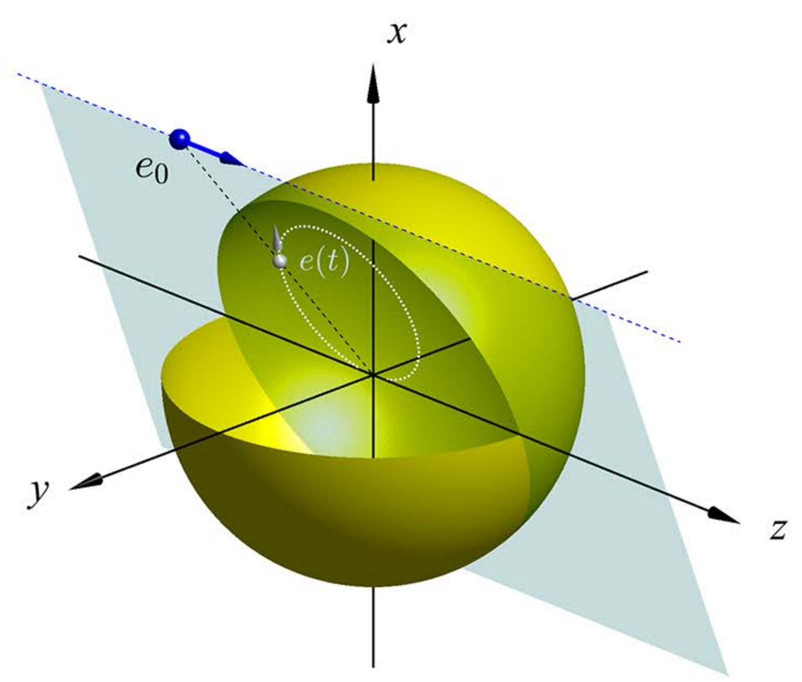

Figure 1: Circular trajectory (white dotted line) of the image charge when a charged particle moves near a conducting sphere.

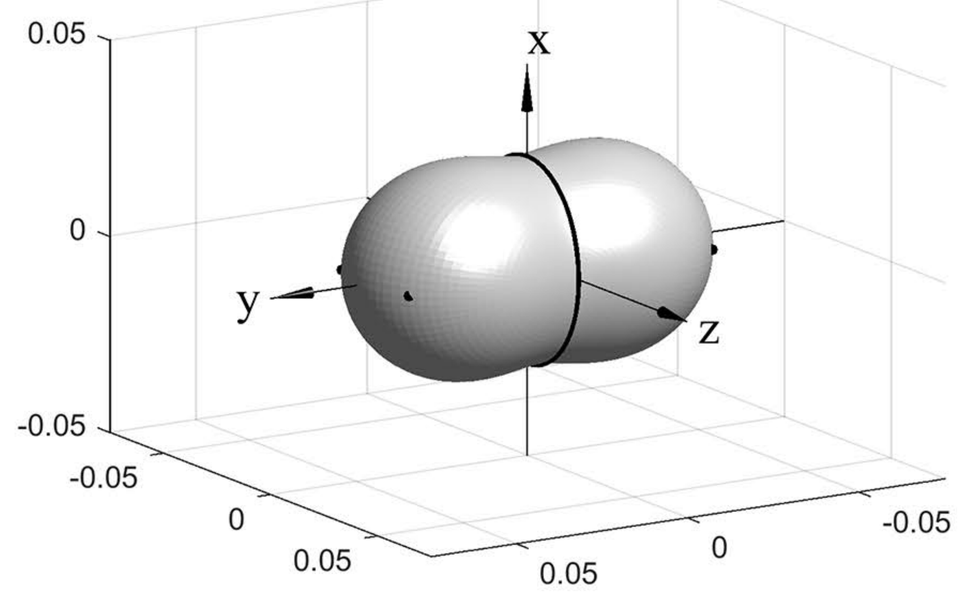

Figure 2: Directional diagram for DR on the sphere at $\mathbf{b} = (R+0, 0)$, $v_0 = 0{,}1c$, and $R\omega/v_0 = 2.34$. Directions of 100% linear polarization are indicated by a bold solid line, and 100% circular polarization are indicated by dots.

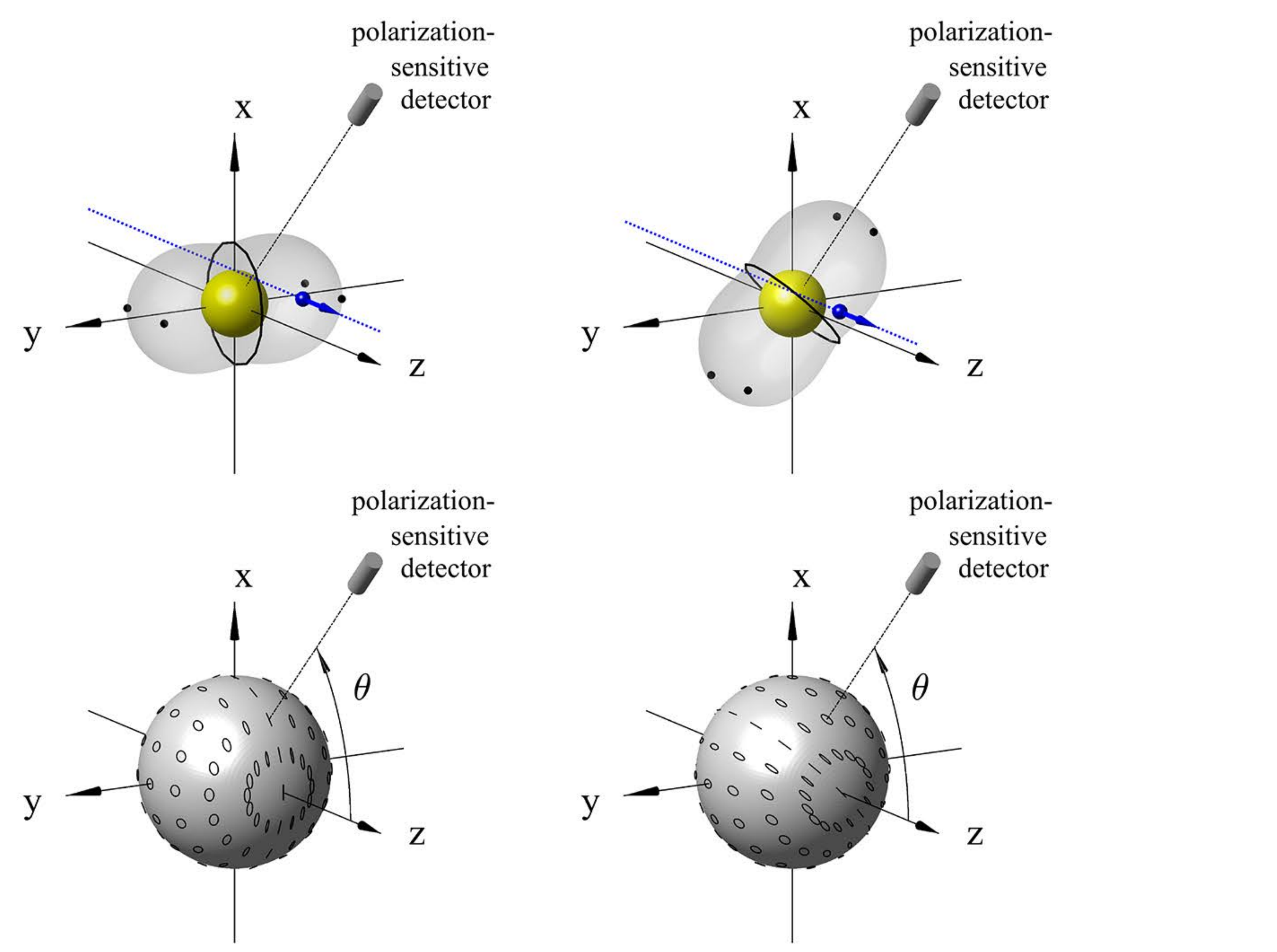

Figure 3: Detecting the polarization of DR at the radiation angle $\theta \neq 0$ makes it possible to determine the relationship between the components x and y of the two-dimensional impact parameter of a particle flying by the sphere. In the case $y = 0$ (left) we have 100% linear polarization, while for $y > 0$ (right) or $y < 0$ we will find elliptical polarization, left or right.

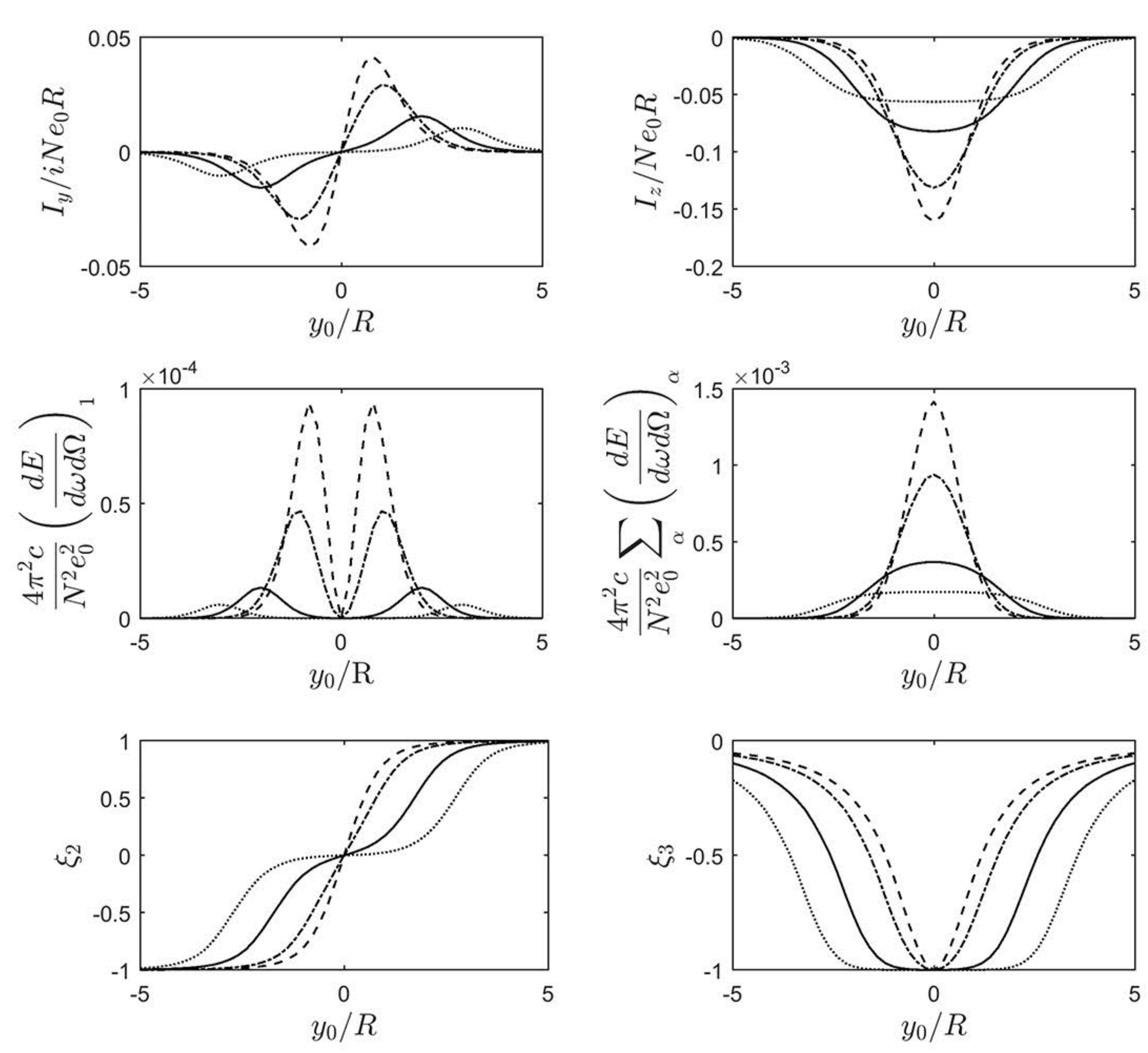

Figure 4: Diffraction radiation characteristics on a sphere in the direction $\theta = \pi/2$, $\varphi = 0$ with dimensions $2a_x = R$, $2a_y = 6R$ (dashed line), $4R$ (solid line), $2R$ (dash-dotted line), R (dashed line). The impact parameter of the bunch center in the direction of the x axis is $x_0 = 1.6R$, the impact parameter of the bunch center in the y direction is plotted on the abscissa axis.

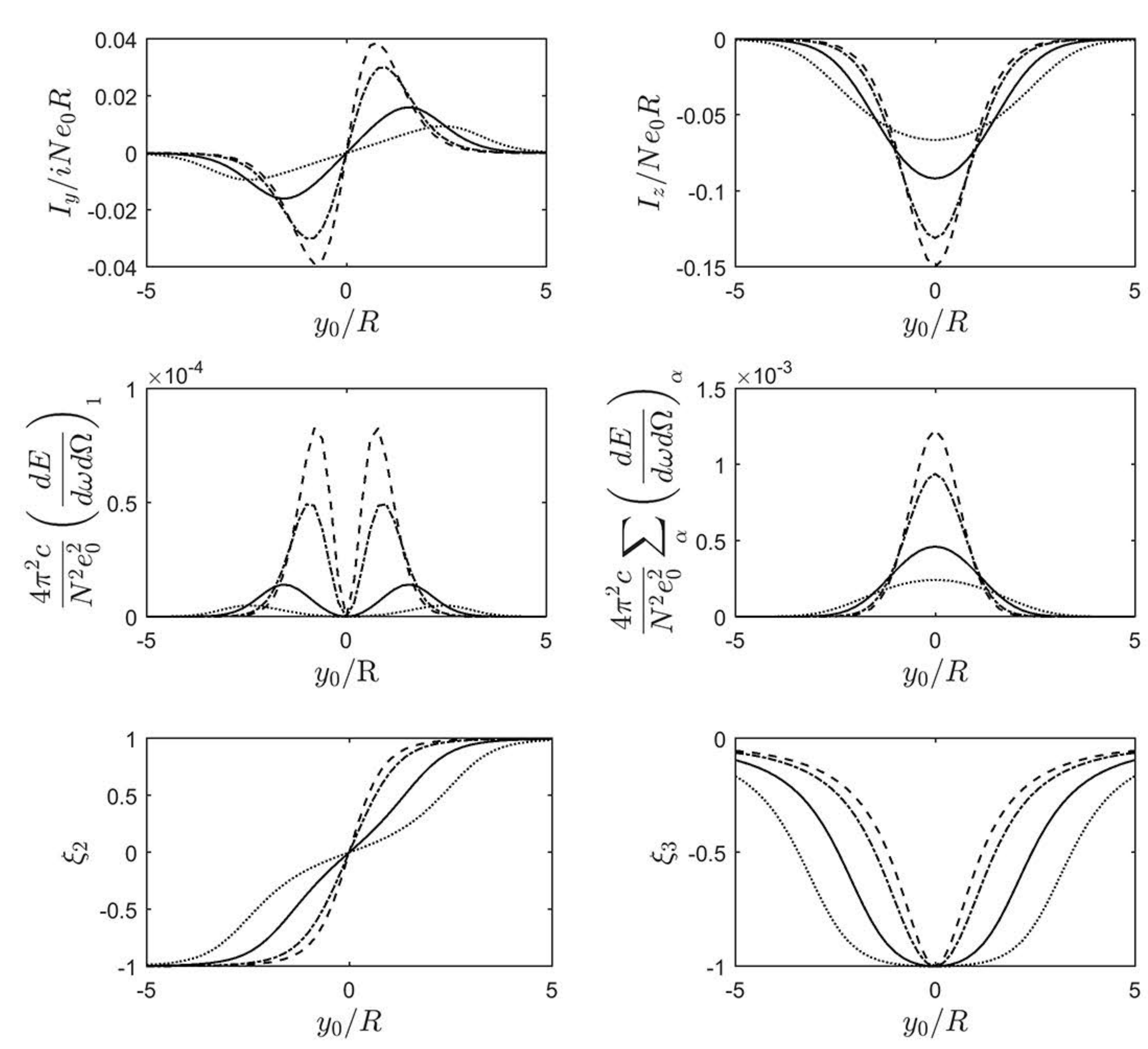

Figure 5: The same for an elliptical bunch, the dimensions of the axes of the ellipse are equal to the sides of the rectangle in the figure 4.