THE THIN STRUCTURE OF WAVEGUIDE DIELECTRIC ACCELERATING SYSTEM ELEMENTS

G.A. Bryzgalov, L.N. Nikolaichuk, N.A. Khizhnyak
National Seintific Center, Kharkov Institute of Physics and Technology,
Akademicheskaya, 1, 61108, Kharkov, Ukraine
E-mail: bryzgalov@kipt.kharkov.ua

Last years the works appeared in which models of artificial dielectrics containing in a frequency band simultaneously negative values of the dielectric permittivity and magnetic permeability were discussed [1, 2]. As is shown in [3], the electromagnetic wave propagation in such a media is characterized with peculiarities that are important to understand the electromagnetic radiation interaction with tissues in vivo.
The aim of this work is to show that such a media in organic nature can be meet at every step, and the mechanism of formation of simultaneous negative values of $\varepsilon(\omega)$ and $\mu(\omega)$ is contained already by their physical structures. Moreover, experimental facts are known confirming the existence of positive and negative values of ε and μ in a narrow frequency band that, unfortunately, is still not completely understood till now.
PACS number: 29.17.+w

1 RESONANCE SCATTERING OF ELECTROMAGNETIC WAVES ON THE DIELECTRIC SPHERE

The problem of electromagnetic wave scattering on the homogeneous dielectric sphere was solved yet at the beginning of the XX-th century [4, 5], but we should use this problem solution in the shape of [6] to investigate the problem of our interest by analytical methods. If the center of a sphere is placed at the origin of coordinates and its radius is denoted with a then for the incident plane linearly polarized electromagnetic wave we have

$$\vec{E}(\vec{r}) = \vec{E}_0 e^{-i\vec{k}_1\vec{r}} \qquad (1)$$

$$\vec{k}_1 = \left(k_1 Sin\theta_0, 0, k_1 Cos\theta_0\right) \qquad (2)$$

and the vector of the electric field intensity $\vec{E}_0$ forms the angle β with the plane $x0z$. Here $k_1 = \frac{\omega}{c}\sqrt{\varepsilon_1\mu_1}$, where ε_1 and μ_1 are the dielectric permittivity and magnetic permeability of the environment, θ_0 is the angle between the vector $\vec{k}_1$ and the axis $0z$.

The scattering field in a wave band has the shape [7, 8]

(the time dependence $e^{i\omega t}$ is omitted), where

$$\vec{E}_{pacc}(\vec{r}) = -i\frac{e^{-ik_1 r}}{k_1 r}\sum_{J=1}^{\infty}\sum_{m=-J}^{+J}\left\{\left[A^{(0)}_{\parallel,J,m}(\theta)Cos(m\varphi-\beta)\right]\vec{e}_\theta + \left[-A^{(0)}_{\perp,J,m}(\theta)Sin(m\varphi-\beta)\right]\vec{e}_\varphi\right\} \qquad (3)$$

$$\vec{H}_{pacc}(\vec{r}) = -\frac{ie^{-ik_1 r}}{k_1 r}\sqrt{\frac{\varepsilon_1}{\mu}}\sum_{J=1}^{\infty}\sum_{m=-J}^{J}\left\{\left[A^{(0)}_{\perp,J,m}(\theta)Sin(m\varphi-\beta)\right]\vec{e}_\theta + \left[A^{(0)}_{\parallel,J,m}(\theta)Cos(m\varphi-\beta)\right]\vec{e}_\varphi\right\} \qquad (4)$$

Here

$$A^{(0)}_{\parallel,J,m}(\theta) = (-1)^{J+m}\sqrt{\frac{\mu_1}{\varepsilon_1}}\sqrt{\frac{(2J+1)(J-m)!}{J(J+1)(J+m)!}}\left[\sqrt{\frac{2J+1}{J}}B^{(0)}_{J,J+1,m}\, mCosec\theta P^{\mu}_J(Cos\theta) + B^{(0)}_{J,J,m}\frac{dP^{\mu}_J(Cos\theta)}{d\theta}\right] \qquad (5)$$

$$A^{(0)}_{\perp,J,m}(\theta) = -(-1)^{J+m}\sqrt{\frac{\mu_1}{\varepsilon_1}}\sqrt{\frac{(2J+1)(J-m)!}{J(J+1)(J+m)!}}\left[\sqrt{\frac{2J+1}{J}}B^{(0)}_{J,J+1,m}\frac{dP^{\mu}_J(Cos\theta)}{d\theta} + B^{(0)}_{J,J,m}\, mCosec\theta P^{\mu}_J(Cos\theta)\right] \qquad (6)$$

are the amplitudes of a scattering wave representing for further investigations the basic interest. Notice that constants $B^{(0)}_{J,J+1,m}$ and $B^{(0)}_{J,J,m}$ in their turn have parameters of the scattering sphere and equal

$$B^{(0)}_{J,J,m} = +i\sqrt{\frac{\varepsilon_1}{\mu_1}}\sqrt{\frac{(2J+1)(J-m)!}{J(J+1)(J+m)!}}\, mCosec\theta_0 Q_j \qquad (7)$$

$$B^{(0)}_{J,J+1,m} = -i\sqrt{\frac{\varepsilon_1}{\mu_1}}\sqrt{\frac{J(J-m)!}{J(J+1)(J+m)!}}\frac{dP^{\mu}_J(Cos\theta)}{d\theta}P_J \qquad (8)$$

$$Q_j = \frac{j'_J(\chi_2) j_J(\chi_1) - \sqrt{\frac{\mu_1 \varepsilon}{\mu \varepsilon_1}} j_J(\chi_2) j'_J(\chi_1)}{j'_J(\chi_2) h_J^{(2)}(\chi_1) - \sqrt{\frac{\mu_1 \varepsilon}{\mu \varepsilon_1}} j_J(\chi_2) h_J^{(2)'}(\chi_1)} \quad (9)$$

$$P_j = \frac{\sqrt{\frac{\mu_1 \varepsilon}{\mu \varepsilon_1}} j'_J(\chi_2) j_J(\chi_1) - j_J(\chi_2) j'_J(\chi_1)}{\sqrt{\frac{\mu_1 \varepsilon}{\mu \varepsilon_1}} j'_J(\chi_2) h_J^{(2)}(\chi_1) - j_J(\chi_2) h_J^{(2)}(\chi_1)} \quad (10)$$

where $\chi_1 = \sqrt{\varepsilon_1 \mu_1} ka$, $\chi_2 = \sqrt{\varepsilon \mu} ka$, $j_J(x)$ and $h_J^{(2)}(x)$ are the spherical radial functions of order J, expressed in terms of the cylindrical Bessel's and Henkel's functions of the half whole index by relations

$$j_J(x) = \sqrt{\frac{\pi}{2x}} J_{J+1/2}(x),\ h_J^{(2)}(x) = \sqrt{\frac{\pi}{2x}} H_{J+1/2}^{(2)}(x), \quad (11)$$

Here ε and μ are the dielectric permittivity and magnetic permeability of the material of the scattering and dielectric sphere manufactured. Multipliers Q_j and P_j have bands on the plane of the complex variable ω, these bands referring to resonance properties of the dielectric sphere corresponding to the dispersion equations were also discussed in literature and nevertheless a number of properties of these equations were not yet studied completely. Note only the denominator of the value of Q_j, expressed by Bessel's cylindrical functions (11) is written as

$$\frac{J_{J-1/2}(\chi_2)}{J_{J+1/2}(\chi_2)} - \sqrt{\frac{\mu_1 \varepsilon}{\mu \varepsilon_1}} \frac{H_{J-1/2}^{(2)}(\chi_1)}{H_{J+1/2}^{(2)}(\chi_1)} = \frac{J}{\chi_1}\left(1 - \frac{\varepsilon}{\varepsilon_1}\right), \quad (12)$$

and the denominator of the value of P_j of the same functions has the shape

$$\frac{J_{J-1/2}(\chi_2)}{J_{J+1/2}(\chi_2)} - \sqrt{\frac{\varepsilon_1 \mu}{\varepsilon \mu_1}} \frac{H_{J-1/2}^{(2)}(\chi_1)}{H_{J+1/2}^{(2)}(\chi_1)} = \frac{J}{\chi_1}\left(1 - \frac{\mu}{\mu_1}\right) \quad (13)$$

Equation (12) is the dispersion equation of TH modes of dielectric sphere oscillations from which $H_p = 0$, and equation (13) is the dispersion equation of TE mode oscillations of the dielectric sphere, respectively, where $E_p = 0$. These equations were investigated by numerical methods in [9].

2 THE AMPLITUDE OF ELECTROMAGNETIC WAVE SCATTERING ON A SMALL DIELECTRIC SPHERE WITH HIGH PERMITTIVITY

If the sphere radius is small $\chi_1 << 1$, then using the Bessel's function asymptotic for small argument values we find that $Q_j \cong \chi_1^{2J+1}$ and $P_j \cong \chi_1^{2J+1}$. Consequently, for small χ_1 only the first items with $J = 1$ play the essential role (3,4). Taking notice, that Bessel's cylindrical functions with a half whole index J are reduced to the trigonometric functions and therefore in case of the small χ_1 and arbitrary χ_2 we find

$$Q_1 = \frac{i\chi_1^3}{3} \frac{1 - 2\frac{\varepsilon}{\varepsilon_1}\frac{Sin\chi_2 - \chi_2 Cos\chi_2}{(\chi_2^2 - 1)Sin\chi_2 + \chi_2 Cos\chi_2}}{\left(1 + \frac{\varepsilon}{\varepsilon_1}\frac{Sin\chi_2 - \chi_2 Cos\chi_2}{(\chi_2^2 - 1)Sin\chi_2 + \chi_2 Cos\chi_2}\right)(1 - i\chi_1) + i\frac{\chi_1^2 (Sin\chi_2 - \chi_2 Cos\chi_2)}{(\chi_2^2 - 1)Sin\chi_2 + \chi_2 Cos\chi_2}} \quad (14)$$

The value of χ_1 in the denominator has been conserved for that the denominator be not converted to zero for any χ_2. If one supposes that in the denominator the items with χ_1 are negligibly small in comparison with the other items then one can write

$$Q_1 = \frac{2i\chi_1^3}{3} \frac{\varepsilon_{eff} - \varepsilon_1}{\varepsilon_{eff} + 2\varepsilon_1} \quad (15)$$

where

$$\varepsilon_{eff} = \varepsilon F(\chi_2) \quad (16)$$

and

$$F(\chi_2) = 2\frac{Sin\chi_2 - \chi_2 Cos\chi_2}{(\chi_2^2 - 1)Sin\chi_2 + \chi_2 Cos\chi_2} \quad (17)$$

and finally, if $\chi_2 << 1$, then $F(\chi_2) \to 1$ and

$$Q_1 = -\frac{2i\chi_1^3}{3} \frac{\varepsilon - \varepsilon_1}{\varepsilon + 2\varepsilon_1} \quad (18)$$

This expression coincides with that of the dipole moment of the dielectric sphere obtained in the long wave approximation. Therefore by analogy with the above approximation one can suppose that the dielectric sphere with a high value of dielectrical permittivity and in any frequency range with respect to electrodynamics is equivalent to the sphere manufactured from dielectrics the effective permittivities of which are equal (16).

In the approximation $\chi_1 << 1$ the value of P_j permits the essential simplification. Really then it has a sense to consider only the item with $J = 1$:

$$P_1 = \frac{i\chi_1^3}{3} \frac{1 - \frac{2\mu}{\mu_1} \frac{Sin\chi_2 - \chi_2 Cos\chi_2}{(\chi_2^2 - 1)Sin\chi_2 + \chi_2 Cos\chi_2}}{\left(1 + \frac{\mu}{\mu_1} \frac{Sin\chi_2 - \chi_2 Cos\chi_2}{(\chi_2^2 - 1)Sin\chi_2 + \chi_2 Cos\chi_2}\right)(1 - i\chi_1) + i\frac{\chi_1^2 (Sin\chi_2 - \chi_2 Cos\chi_2)}{(\chi_2^2 - 1)Sin\chi_2 + \chi_2 Cos\chi_2}} \quad (19)$$

In case of $\chi_1 << \chi_2$ we find that

$$P_1 = -\frac{2}{3} i\chi_1^3 \frac{\mu_{eff} - 1}{\mu_{eff} + 2} \quad (20)$$

where

$$\mu_{eff} = \mu F(\chi_2) \quad (21)$$

The function $F(\chi_2)$ in this case turns to the same one as for the introduction of the effective value of permittivity ε_{eff} (17).

In the limiting case $\chi_1 << 1$ and $\chi_2 << 1$, we find

$$P_1 = -\frac{2}{3} i\chi_1^3 \frac{\mu - \mu_1}{\mu + 2\mu_1} \quad (22)$$

that coincides with the magnetic dipole moment obtained for the magnetic sphere. It permits to suppose that in the high-frequency region when $F(\chi_2)$ can accept both positive and negative values of Q_1 and P_1, despite on the smallness the parameter χ_1 can be high due to the smallness of denominators

$$\varepsilon_{eff} + 2\varepsilon_1 \to 0 \quad (23)$$

$$\mu_{eff} + 2\mu_1 \to 0 \quad (24)$$

The condition (23) determines the condition of arising the electrical resonance and the condition (24) – refers to the magnetic one of the sphere.

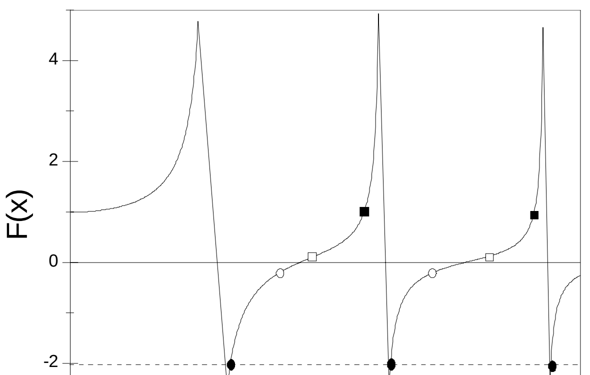

Fig. 1. The graph of the function $F(\chi)$. The points refer to resonances of the dielectric sphere:

● - *magnetic resonances,* $F(\chi) = -2$;

○ - *electrical resonances,* $F(\chi) = -\frac{2\varepsilon_1}{\varepsilon}$;

□ - *electrical resonances of passing,* $F(\chi) = \frac{\varepsilon_1}{\varepsilon}$;

■ - *magnetic resonances of passing,* $F(\chi) = 1$.

Thus resonance conditions depend essentially on geometric sphere dimensions then resonances themselves were called as geometric ones.

The graph of the function $F(\chi)$ is shown in Fig. 1 and it shows effective values of the dielectric permittivity and magnetic permeability of the sphere material and above mentioned graph was firstly defined in [10], and by rigorous electrodynamical calculations in [7, 8].

3 THE EXPERIMENTAL INVESTIGATION OF ELECTROMAGNETIC WAVE SCATTERING ON DIELECTRIC SPHERES WITH HIGH VALUES OF THE DIELECTRIC PERMITTIVITY

In the process of manufacturin articles from titanium dioxide (natural TiO_2, crystal grinding, mixing, and successive fabrication from this mixture and a binder of articles) it was necessary to control the quality of isotropic dielectrics made on their basis. For this aim probes, from which one fabricated specimens of a spherical form, were taken from different places of the mix and resonance properties were studied in the 10-sm range of wavelengths. Resonance frequencies were calculated according to relations (16) and (21) and compared with experimental measurements [11, 12].

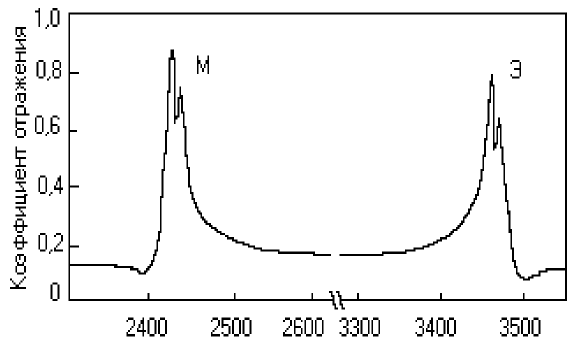

Fig. 2. The graph of the dependence of the reflection factor on the frequency of the dielectric sphere from TiO_2, with 13.20 mm in diameter placed in the center of the rectangular waveguide.

Typical experimental curves are shown in Fig. 2 for magnetic and dielectric oscillations in the dielectric sphere. In case of electrical oscillations the resonance curves are similar to magnetic ones but the minimum is on the left of the maximum of the reflection factor.

The position of resonance was used to precise measurements of high values of ε in a wide range of frequencies and temperatures. However in these experiments resonance splitting even for spheres of a small radius and the position minimum values of the reflection factor have not been explained. These peculiarities become obvious if to take account that for definite values of $F(\chi)$ ε and μ are negative. Indeed in these cases the electrical dipole moment of a sphere (15) refers to the wave reflection factor and behaves itself as in (Fig. 1). Therefore under conditions of resonance (23) the dipole momentum passes through zero in the negative part that reduces to the maximum splitting shown in Fig. 2 (η is dependent on the dipole momentum modulus).

Further the dipole momentum has its turn again to the region of positive values and when it is zero the reflection factor is again zero. On the graph of Fig. 1 characteristic points of the reflection factor value are shown for $F(\chi)$ as a function of frequency. Black points show the position of magnetic resonances and values for which the minimum is defined and light points relate to the position of electrical resonance's in the region of negative $F(\chi)$ and the position of minima of reflections for positive values of $F(\chi)$.

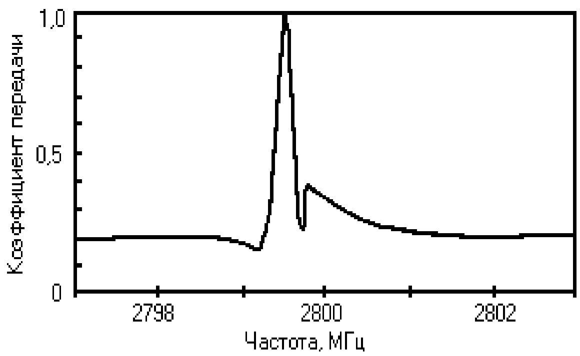

Fig. 3. Transfer ratio versus the frequency of a single dielectric disc from TiO_2 of the thickness of 3.291 mm, with the central hole of 5 mm in diameter in a circular half endless waveguide of 81.5 mm in inner diameter E_{01} oscillations have been excited.

The analogous relation is observed for more massive dielectric specimens. In the graph of Fig. 3 the dependence of the gain factor is presented (the value opposite to the reflection factor) for the dielectric disc overlapping the section of a circular waveguide. As in case of a sphere a signal has a minimum determining wave passing and then transition of the dielectric permittivity of the disk as a whole one to the negative region in the place of maximum splitting (on modulus). This value refers to positive ones and after deviation from the resonance value turns to the average value.

The value of ε_{eff} is defined according to the frequency of resonance curve splitting. This value of ε_{eff} was used to calculate the waveguide dielectric resonance accelerating structure on the basis of the circular waveguide loaded by dielectric discs and was confirmed experimentally [12, 13].

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