NEW METHOD OF ANALYZING WAVE PROCESSES IN PULSE GENERATORS BASED ON LINES WITH DISTRIBUTED PARAMETERS

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A new method of theoretical analysis of wave processes in high-current pulse generators through the relations between integral values reflecting regularities of energy transfer in ideal lines with distributed parameters is described. The use of the method developed considerably simplifies the procedure of searching for an optimal - from the point of view of getting maximal efficiency – relation of impedances for pulse facilities on stepped lines including those with arbitrary number of cascades. High efficiency of the method is demonstrated by several examples.
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1 INTRODUCTION

A search for new circuits for multi-cascade pulse generators is performed, as a rule, by two stages. Firstly, a generator circuit is selected. Then transient processes are analyzed basing on the subsequent consideration of voltage or current waves propagating in lines. As a result, a mathematical expression for the voltage and load current, involving impedances of all cascades can be obtained. Impedances are optimized to get a maximum efficiency, voltage or current. A complexity of traditional approach is conditioned by the necessity of accounting a large number of wave limits in the circuits before such an analysis. As a rule, these are circuits with a number of cascades not greater than 2 or 3.

In the course of fundamental development of generators based on stepped lines [1-4] the author developed a new method of search for an optimal relation of impedances to reach 100% efficiency. Below, there are contemplated some regularities of energy transfer in generators with transmission lines the account of which in some cases significantly simplifies a search for optimal impedances. The method application is illustrated on the example of generators with different techniques of energy storage. All generators are formed by the homogeneous lines (cascades) of the equal electrical length T_0. For each case there are defined conditions when for idealized models the generator has 100% efficiency at formation of a squared pulse of $2T_0$ duration on the matched load.

2 SOME REGULARITIES OF ENERGY TRANSMISSION IN IDEAL LINES

From Maxwell equations one can get two integral relationships. The first one determines a relation between time integral of voltage for arbitrary closed circuit L and a change of magnetic induction flux through an arbitrary surface S, which is supported by this circuit: $\int_0^t U dt = \Phi_{ms}(0) - \Phi_{ms}(t)$, where $U = \oint_L \vec{E} d\vec{l}$. The second relationship defines a relation of time integral of conduction current flowing through a closed surface S, with a charge change in the volume V limited by this surface: $\int_0^t I dt = q(0) - q(t)$.

Provided the energy is extracted out from the volume V by the time moment t_0, the formulas have a simpler form:

$$\begin{cases} \int_0^{t_0} U dt = \Phi_m(0), & (1) \\ \int_0^{t_0} I dt = q(0). & (2) \end{cases}$$

Following equations (1), (2) is a necessary and sufficient condition for a complete extraction of energy. It is convenient to select the time moment as a beginning of the integration interval $t = 0$, when, as a result of commutation, in the generator electromagnetic waves appear. In this case for generators with a capacitive energy storage the right part of equation (1) is equal to zero, and for generators with an inductive energy storage - the right part of equation (2) equals to zero.

The formulas obtained do not find a wide application when solving electrotechnical problems, as $U(t)$ and $I(t)$ in most cases change by unknown law that makes impossible their integration. The situation is different for stepped lines, when pulses are in the form of squared steps, and time integration amounts to summation of constant magnitude products. The time integral of voltage equaling to zero, as a necessary condition for a complete energy extraction from capacitive generators, was mentioned earlier in paper [5].

3 CAPACITIVE GENERATOR

Let us consider as an example a circuit of a generator (Fig. 1) formed from $n \geq 2$ cascades with impedances $Z_1, Z_2, ..., Z_n$. Cascades with impedances Z_{n-1}, Z_n are charged from the external source up to the voltage U_0. After charging is finished ($t = 0$), a switch S_1 is closed. The load $Z_L = Z_1$ is connected to the generator by a switch S_2 ($t = nT_0$) with delay by $2T_0$ with regard to arrival of the first wave from switch S_1 to it.

As the voltage along the circuit L (Fig. 1a) differs from zero only in the cross-section AB, equation (1) takes the following form:

$$\int_0^{t_0} U_{AB}dt = 0 . \quad (3)$$

As cascades with numbers $1 \div (n-2)$ are not initially charged, for the surface S (Fig. 1a) intersecting a grounding electrode near the load and at the juncture of i and $i+1$ cascades, equation (2) is converted into the form:

$$\int_0^{t_0} Idt = 0 . \quad (4)$$

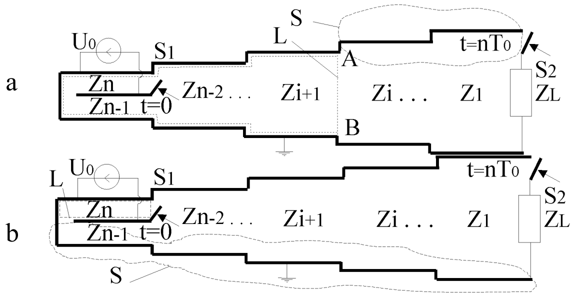

Fig. 1. Capacitive generator circuit.

Let the wave arrives to the cross-section AB at the time moment t_i. The voltage and current in this cross-section should differ from zero in the time interval $t_i \div (t_i + 4T_0)$. If one designates the amplitude of voltage and current in the time intervals $t_i \div (t_i + 2T_0)$ and $(t_i + 2T_0) \div (t_i + 4T_0)$ as U_{i1}, I_{i1} and U_{i2}, I_{i2}, equations (3), (4) takes the form $U_{i1} + U_{i2} = 0$, $I_{i1} + I_{i2} = I_L = U_L / Z_1$. Energy transmitted through the cross-section AB should be equal to the energy absorbed in the load: $(U_{i1}I_{i1} + U_{i2}I_{i2}) \cdot 2T_0 = (U_L^2 / Z_1) \cdot 2T_0$. When solving jointly equations with the use of relations $U_{i1} / I_{i1} = Z_i$ and $U_{i-1,1} = U_{i1} 2Z_{1-1} / (Z_{i-1} + Z_i)$ we obtain:

$$2U_{i1}^2 / Z_i + U_{i1}U_L / Z_1 = U_L^2 / Z_1 . \quad (5)$$

After subtraction from (5) of the similar equation for the voltage $U_{i-1,2}$ with regard to $U_L = -2U_{11}$ let us find the final expression for relation of impedances of cascades with numbers $i \le n-2$:

$$Z_i = 2Z_1 / [i \cdot (i+1)] \quad (6)$$

In this case the voltage on the load is

$$U_L = -2U_{n-2,1} \prod_{j=1}^{n-3} [2Z_j / (Z_j + Z_{j+1})] = -U_{n-2,1}(n-1) . \quad (7)$$

Now let us line a circuit (Fig. 1b) passing along the grounding and high-voltage electrodes of a line with impedance Z_n through switch S_1 and junction of lines with impedances Z_n, Z_{n-1}. When closing the switch S_1 the voltage along the circuit differs from zero only at the junction of cascades with impedances Z_n, Z_{n-1}. One can demonstrate that in the time interval $0 \div T_0$ the voltage is U_0 and in the interval $T_0 \div 3T_0$ - $U = -U_0 \{(Z_{n-1} - Z_n) / (Z_{n-1} + Z_n) + [2Z_{n-1}Z_{n-2} / (Z_{n-1} + Z_{n-2}) \cdot (Z_{n-1} + Z_n)]\}$. Further the voltage in this cross-section should be equal to zero, for, otherwise, the residual energy will not have a chance to arrive at the load by the time moment $t = (n+2)T_0$. From the condition of equation to zero of time integral of voltage in this cross-section (1) we obtain:

$$(Z_{n-1} - Z_n) / (Z_{n-1} + Z_n) + [2Z_{n-1}Z_n / (Z_{n-1} + Z_{n-2})(Z_{n-1} + Z_n)] = 1/2 . \quad (8)$$

Through the left bound into the volume limited by the surface S (Fig. 1b) in the time interval $T_0 \div 3T_0$ there flows out current $U_0 / (2Z_n)$, and through the left bound in the time interval $nT_0 \div (n+2)T_0$ there flows out current U_L / Z_1. The voltage on the load can be determined with regard to (7): $U_L = U_0(n-1)Z_{n-2} / (Z_{n-1} + Z_{n-2})$. As for the case under study $q(0) = U_0 T_0 / Z_{n-1}$, equation (4) is transformed as:

$$1/Z_{n-1} = 1/Z_n - 2(n-1) \cdot Z_{n-2} / [Z_1 \cdot (Z_{n-1} + Z_{n-2})] . \quad (9)$$

Solving equations (8), (9) with regard to (6) we find: $Z_n = 2Z_1 / [n(n+1)]$, $Z_{n-1} = 2Z_1 / [(n-1)n]$.

In ideal case such a generator possesses 100% efficiency and forms a rectangular voltage pulse of $2T_0$ duration and amplitude $U_L = nU_0 / 2$ on the matched load. Addition of each supplementary cascade to the generator raises the voltage by $U_0 / 2$.

4 INDUCTIVE GENERATOR

Let us consider a stepped forming line (SFL, Fig. 2) constituted by n subsequently connected cascades. In the initially closed circuit composed by electrodes of SFL and current opening switches S_1 and S_2 under the action of the external source the current is created I_0 and magnetic energy is stored in the SFL. At the time moment $t = 0$ the opening switch S_1 disconnects the current source, and the load is connected to the stepped forming line at S_2 operation at the time moment $t = nT_0$.

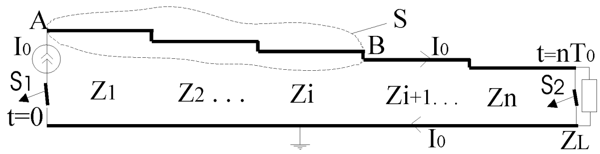

Fig. 2. Inductive generator circuit.

For the closed surface S covering a part of one of electrodes of SFL from the output (point A) to the juncture of i and $i+1$ cascades (point B), according to (2) for circuits with an inductive energy storage $\int_0^{t_0} Idt = 0$. As at $t \ge 0$ S_1 is opened, the value I represents the current at the juncture of i and $i+1$ cascades. Before the current arrival from S_1 the current I_B equals I_0. At the time moment $t = iT_0$ the current becomes equal $I_0\{1-2^i\cdot\prod_{i=2}^{i+1}[Z_{i-1}/(Z_{i-1}+Z_i)]\}$ and remains constant in the time interval $iT_0 \div (i+2)T_0$. At $t>(i+2)T_0$ the current $I_B=0$. Equating the I_B current time integral to zero we obtain $t=(n-i-2)T_0$ or $\prod_{j=2}^{i+1}\left[Z_{j-1}/\left(Z_{j-1}+Z_j\right)\right]=(i+2)\cdot 2^{-(i+1)}$.

The given equation is reduced to the form:

$$Z_i/Z_1 = 2/[i(i+1)] \quad (10)$$

Provided that (10) is adhered, on the matched load there is formed a rectangular current pulse of the amplitude $I_0 n/2$ and duration $2T_0$, during this pulse the energy is fully delivered to the load.

5 INDUCTIVE-CAPACITIVE GENERATOR

The generator (Fig. 3) contains $n\geq 2$ cascades with impedances $Z_1, Z_2, ..., Z_n$. The magnetic energy W_{L0} is stored in all cascades under the action of the I_0 current, formed by the external source. Simultaneously, the electric energy W_{C0} is stored in $n-1$ and n cascades charged up to the voltage U_0 from another source. At the time moment $t=0$ the switch S_1 is closed and opening switch S_2 connects the load at the time moment $t=(n-2)T_0$. Switch S_3 must close the grounding electrode before the first wave arrives from S_1.

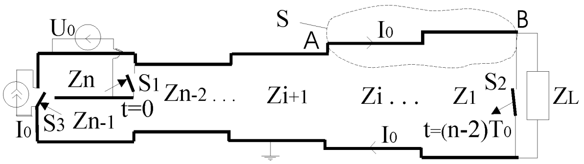

Fig. 3. Inductive-capacitive generator circuit.

Applying the integral equation (1) for the circuit passing through a short-circuited electrode of stepped lines, supposing that the voltage on the load is U_L we obtain:

$$U_L = (I_0 L)/2T_0 = (I_0\cdot\sum_{i=1}^{n} Z_i)/2T_0, \quad (11)$$

where L is a total inductance of stepped line. The energy transmitted to the load with regard to (11) is

$$W_L = U_L I_L 2T_0 = 0.5 I_L I_0 T_0 \sum_{i=1}^{n} Z_i. \quad (12)$$

As $W_L = W_{L0}+W_{C0}$ we get:

$$I_L/I_0 = 0.5\left(1+W_{C0}/W_{L0}\right) = 0.5(1+\lambda), \quad (13)$$

where the factor $\lambda = W_{C0}/W_{L0}$.

Let us now consider the surface S (Fig. 3). Current flowing through the surface S at the point A in the time $0\div(n-i-2)T_0$ is equal to I_0. After the arrival of the voltage wave of U_i amplitude at the i cascade from S_1 at the time moment $t=(n-i-2)T_0$ the current increases up to I_0+U_i/Z_i and remains constant during the time interval $2T_0$. Beginning with the time moment $t=(n-i)T_0$ the current in this cross-section must be equal to zero. The current flowing out through the surface S at the point B remains equal to I_0 until the switch S_2 is opened. Then in the time interval $(n-2)T_0 \div nT_0$ this current equals to the load current U_L/Z_1.

Taking into account that $q_0=0$ the relation (2) will be written in the form

$$(I_0+U_i/Z_i)2T_0 = I_0 i T_0 + U_L 2T_0/Z_1$$

or

$$2U_i/Z_i = I_i\cdot(i-2)+2U_L/Z_1 \quad (14)$$

Taking into account that

$$U_1 = U_i\cdot\prod_{j=1}^{i-1}[2Z_j/(Z_j+Z_{j+1})], \quad (15)$$

$$U_L = U_1 + I_0\cdot Z_1/2, \quad (16)$$

from (14) one can get a relation

$$2\left(U_L - I_0 Z_1/2\right)/Z_i =$$

$$=\left[I_0(i-2)+2U_L/Z_i\right]\cdot\prod_{j=1}^{i-1}[2Z_j/(Z_j+Z_{j+1})].$$

Solving this equation together with a similar equation written for a cascade with a number greater by one unit we find a recurrence formula $Z_{i+1}=Z_i[2I_L/I_0+$ $+(i-2)]/(2I_L/I_0+i)$ and an expression for impedances of cascades with numbers $i=1\div(n-2)$:

$$Z_i = Z_1\lambda(\lambda+1)/[(\lambda+i)\cdot(\lambda+i-1)] \quad (17)$$

After the switch S_1 is closed the energy must be fully extracted from the cascade with the impedance Z_n in the time interval $0\div T_0$. In the mathematical form this condition has the form:

$$U_0 = Z_n\cdot I_0 \quad (18)$$

To avoid a recharge of this cascade one must avoid appearance of the voltage wave reflected from the juncture of n and $n-1$ cascades:

$$U_0\left(Z_{n-1}-Z_n\right)/\left(Z_{n-1}+Z_n\right)+U_0 2Z_n Z_{n-1}/$$
$$[\left(Z_n+z_{n-1}\right)\left(Z_{n-1}+Z_{n-2}\right)]=0 \quad (19)$$

When closing the switch S_1 the voltage wave will go into cascade $n-2$

$$U_{n-2} = U_0 Z_{n-2}/\left(Z_{n-1}+Z_{n-2}\right). \quad (20)$$

The amplitude of this wave can be also determined from equation (15) taking into account (16), (17):

$$U_{n-2} = U_1/\prod_{j=1}^{n-3}\left[2Z_j/\left(Z_j+Z_{j+1}\right)\right] = 0.5\cdot\left(2I_L-I_0\right)/$$

$$\prod_{j=1}^{n-3}\left[(k+j)/(k+j-1)\right] = 0.5 Z_1 I_0\lambda(\lambda+1)/(\lambda+n-2). \quad (21)$$

Equating the right sides of (20) and (21) we obtain: $U_0 Z_{n-2}/\left(Z_{n-1}+Z_{n-2}\right) = 0.5 Z_1 I_0 \lambda(\lambda+1)/(\lambda+n-2)$.

From (19) we find:

$$Z_n = Z_{n-1}(Z_{n-2}+Z_{n-1})/(Z_{n-2}-Z_{n-1}). \quad (22)$$

Substituting the expression for Z_n in (21) with regard to (18) we find the impedance Z_{n-1}:

$$Z_{n-1} = Z_1\lambda(\lambda+1)/[(\lambda+n-2)\cdot(\lambda+n-1)]. \quad (23)$$

The optimal impedance Z_n is found after substitution of the expression (23) in (22):

$$Z_n = Z_1\lambda(\lambda+1)/(\lambda+n-1) . \quad (24)$$

The voltage on the load is equal to

$$U_l = U_0(\lambda+n-1)/2\lambda . \quad (25)$$

The optimal relation of generator impedances (Fig. 3) is determined by equations (17), (23) and (24). Besides, there should be matched the amplitudes of U_0 and I_0 according to expression (18). Polarity of charged voltage should be so that after the switch S_1 is closed, the current in the line n decreases.

6 CONCLUSION

There was developed a new method of calculation, whose employment significantly simplifies a search for optimal relation of impedances of multi-cascade generators on stepped lines possessing in the ideal case 100% efficiency. A high efficiency of the method was demonstrated on the example of generator circuits with different methods of energy storage.

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