BEAM25 PROGRAM FOR NUMERICAL SIMULATION OF THE PROCESSES OF HIGH-CURRENT ELECTRON BEAM SHAPING, ACCELERATION AND TRANSPORTATION

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The paper presents the description of a self-consistent difference scheme and numerical algorithm of a joint solution of Maxwell equations and equation of charged particle motion on a regular non-uniform mesh. The given algorithm is applied to calculate non-stationary electromagnetic processes in axially symmetric structures consisting of components with different electric and magnetic properties as well as to simulate numerically the processes of high-current electron beam shaping, acceleration and transportation. The example of the program use for the calculation of electron beam characteristics in a STRAUS-2 accelerator diode is given. The obtained coincidence of calculation and experimental data lies within the limits of ±10%.
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1 INTRODUCTION

Over the course of the last several years VNIIEF developed iron-free linear induction accelerators (LIA) [1-3], taking advantage of inductors with water insulation on the basis of lines with distributed parameters as basic accelerating system components. When creating such facilities one should solve a number of problems. To those most significant one can relate the development of systems aimed at the high-voltage accelerating voltage formation for the injector and inductors with the required electric characteristics as well as the problems of forming, accelerating and transporting a high-current electron beam. In the present paper we develop - on the basis of solving the system of Maxwell equations and equation of motion - the algorithm of numeric simulation of non-stationary electromagnetic processes in axially symmetric structures consisting of components with different electric properties as well as of the processes of high-current electron beam formation, acceleration and transportation. The example of using the program for calculation of electron beam characteristics in the diode of STRAUS-2 accelerator is given [4]. The comparison of calculation results with the experimental data is presented as well.

2 TASK SETUP

A beam of charged particles is simulated in the form of macro-particles whose motion in self-consistent electromagnetic fields is described by the following formula:

$$\frac{d\vec{p}}{dt} = e\left(\vec{E} + \left[\vec{v} \times \vec{B}\right]\right) \tag{1}$$

The electric and magnetic fields meet the Maxwell equations:

$$rot\vec{H} = \frac{\partial \vec{D}}{\partial t} + \vec{j} \,,\ rot\vec{E} = -\frac{\partial \vec{B}}{\partial t} \,,\ div\vec{D} = \rho \,,\ div\vec{B} = 0 \,, \tag{2}$$

where $\vec{p}$ and $\vec{v}$ - particle pulse and velocity, $\vec{E}$ and $\vec{D}$ - electric field strength and inductance; $\vec{H}$ and $\vec{B}$ - magnetic field strength and inductance, $\vec{j}$ - current density, moreover, $\vec{j} = \vec{j}_{ext} + \vec{j}_{cond}$, where $\vec{j}_{cond}$ is calculated in the media with the known conductivity while $\vec{j}_{ext}$ - in other cases. Such presentation of the current density is made for the sake of convenient realization of the calculation program. Fields and conductivity currents are related through the following additional relations:

$$\vec{D} = \varepsilon\varepsilon_0\vec{E} \,,\ \vec{B} = \mu\mu_0\vec{H} \,,\ \vec{j}_{cond} = \sigma\vec{E} \,,$$

where ε_0, μ_0 - dielectric and magnetic permittivities of vacuum, ε, μ - relative dielectric and magnetic permittivities of material media, σ - conductance.

The initial conditions for the components of electromagnetic fields and distribution of charge and current densities in the given program version are the following:

$$\vec{E}(\vec{x},0) = \vec{E}_0(\vec{x}) \,,\ \vec{B}(\vec{x},0) = 0 \,,\ \rho(\vec{x},0) = 0 \,,\ j(\vec{x},0) = 0 \,.$$

The initial distribution of electric field is found from the solution of the equation $\Delta\phi = 0$ [5], where ϕ is the electric field potential.

The boundary conditions finalize the setup of the task and are formulated in the following way:
- surface of metal: $E_\tau = 0$, $B_n = 0$.
- interface of media with different electric and magnetic properties:

$$E_{1\tau} = E_{2\tau} \,,\ D_{2n} - D_{1n} = \rho_s \,,\ H_{2\tau} - H_{1\tau} = j_s \,,\ B_{1n} = B_{2n} \,,$$

where ρ_s - surface density of free charges, j_s – linear density of surface current.
- axis of symmetry:

$$E_{r,\theta} = 0 \,,\ B_{r,\theta} = 0 \,;$$

- open boundary with radiation condition (input and output TM and TE waves):

$$\sqrt{\varepsilon\varepsilon_0}\,E_\tau \pm \sqrt{\mu\mu_0}\,H_\theta = 0 \,,\ \sqrt{\varepsilon\varepsilon_0}\,E_\theta \pm \sqrt{\mu\mu_0}\,H_\tau = 0 \,,$$

where n and τ indexes imply normal and tangential components of the field.

Boundary conditions for the particles consist in the fact that when the particle crosses metal interfaces and media interfaces with different electric properties it is absorbed.

3 DIFFERENCE SCHEME OF SOLVING MAXWELL EQUATIONS AND EQUATION OF MOTION

Maxwell equations (2) are solved basing on the rectangular non-uniform mesh (Fig. 1). To solve them there is used a difference-overstepping scheme with the following accuracy: $O(\Delta t^2 + \Delta r^2 + \Delta z^2)$. The equation is approximated by central differences by each coordinate (r, θ, z) in terms of axial symmetry $(\partial /\partial \theta = 0)$ in the following way [5-7]:

$$\left(\frac{\Delta B_r}{\Delta t}\right)^n = \frac{\Delta E_\theta^n}{\Delta t}, \qquad (3)$$

$$\left(\frac{\Delta B_\theta}{\Delta t}\right)^n = \left(\frac{\Delta E_z^n}{\Delta r} - \frac{\Delta E_r^n}{\Delta z}\right), \qquad (4)$$

$$\left(\frac{\Delta B_z}{\Delta t}\right)^n = -\frac{1}{r_{(j+0.5)}} \cdot \frac{\Delta(rE_\theta^n)}{\Delta r}, \qquad (5)$$

$$\left(\frac{\Delta D_r}{\Delta t}\right)^{n+0.5} = -\frac{\Delta H_\theta^{n+0.5}}{\Delta z} - \sigma E_r^{(n+0.5)} - j_r^{n+0.5}, \qquad (6)$$

$$\left(\frac{\Delta D_\theta}{\Delta t}\right)^{n+0.5} = \frac{\Delta H_r^{n+0.5}}{\Delta z} - \frac{\Delta H_z^{n+0.5}}{\Delta r} - \sigma E_\theta^{(n+0.5)} - j_\theta^{n+0.5}, \qquad (7)$$

$$\left(\frac{\Delta D_z}{\Delta t}\right)^{n+0.5} = \frac{1}{r_j}\frac{\Delta(rH_\theta^{n+0.5})}{\Delta r} - \sigma E_z^{(n+0.5)} - j_z^{n+0.5}. \qquad (8)$$

Here Δt is a step by time, n index – number of the time layer, magnetic fields and current densities are defined on half-full $(n+0.5)$ time layer, electric fields - on full time layer (n). Indexes i and j correspond to the numbers of nodes by z and r, while $i+0.5$ and $j+0.5$ – to the centers of cells by z and r. The $(n+0.5)$ operator implies a half-sum by time, for example, $E_z^{(n+0.5)} = (E_z^{n+1} + E_z^n)/2$. By parity of reasoning, $(j+0.5)$ is a half-sum by r, while $(i+0.5)$ – by z.

The equation of motion is integrated by the method of overstepping [5, 6]:

$$\frac{\vec{p}'^{n+0.5} - \vec{p}'^{n-0.5}}{\Delta t} = \frac{e}{m}\left(\vec{E}^n + [\vec{v}^{(n)} \times \vec{B}^{(n)}]\right), \quad (9)$$

$$\frac{\vec{x}^{n+1} - \vec{x}^n}{\Delta t} = \vec{v}^{n+0.5}, \qquad (10)$$

where $\vec{p}' = \vec{v}\gamma$ the reduced pulse of the particle, $\gamma = (1 - v^2/c^2)^{-0.5}$, v - particle velocity, e and m – particle charge and mass, $\vec{x}$ - particle radius vector. When solving the equation of motion there are taken into account two components of coordinates (r, z) and three components of velocity (v_r, v_θ, v_z). The integration of equation (9) is performed by the Boris algorithm [5, 6] that uses separation of electric and magnetic forces. The distribution of the particle charge over the nodes of the space mesh for calculation of charge and current densities as well as the determination of the force affecting the particle is implemented with the aid of the CIC model (cloud in cell) [5, 8]. In this model the particles represent rectangles with a uniform density of the charge. The lengths of rectangle sides are not connected with the steps of the space mesh by any relation. The contribution of the charged particle to the nodes of the grating is proportional to the share of the area fallen to the cell surrounding the corresponding node.

The microscopic inconsistency of densities of currents and charges caused by the use of meshes and distribution of charges and currents leads to the non-observation of the charge conservation law in the difference form $\frac{\partial \rho}{\partial t} + div\vec{j} = 0$. To make the equation satisfied the correction of densities of currents and charges is performed on the basis of solving the continuity equation [5, 6].

Scheme (3-8) is stable at realizing the inequality for Δt integration step:

$$\Delta t < \tau_{min} = min(\tau_{mesh}),$$

where the value of τ_{mesh} is found from equation [6, 9]:

$$(1 - \sigma\tau_{mesh}) = \sqrt{\varepsilon\mu}\tau_{mesh}^2 (h_r^{-2} + h_z^{-2}),$$

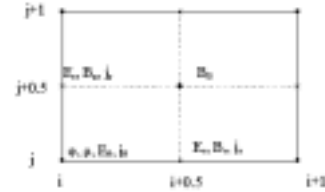here h_r and h_z are the sizes of cells by r and z, respectively.

Fig. 1. Fragment of space mesh with field components.

The described scheme of solving Maxwell equations and equation of particle motion is realized in BEAM25 program written in Fortran.

4 CALCULATION RESULTS

With the aid of BEAM25 program there were calculated the processes of electromagnetic fields and electron flux formation in the pulse electron accelerator STRAUS-2 [4], the calculated geometry of which is presented in Fig. 2 along with the beam space distribution 9. The accelerator includes a system of high-voltage pulse formation 1 and an accelerating tube with a diode assembly. The forming system is produced on the basis of a double step forming line (DSFL).

DSFL consists of five sections of uniform lines of an electric length approximately equal to 18 ns with impedances optimized for getting a maximal voltage transformation ratio. DSFL switching is implemented with the aid of a multi-channel switch formed by 20 trigatron-type gas-filled gaps 2 distributed uniformly by radius. The first voltage pulse of a positive polarity is cut by a pre-pulse gap 3. The second working pulse of a negative polarity arrives at a diode where the pulse of the electron beam current is formed.

The accelerating tube is formed by a polyethylene tube 4 and sectioned insulator 6 with the electrolyte layer serving for equalizing the electric potential distribution by the insulator length between them. Cathode 7 is displaced in the internal vacuum volume of the accelerating tube. The diode assembly is separated of the forming system with a polyethylene diaphragm the volume between it and accelerating tube being filled with
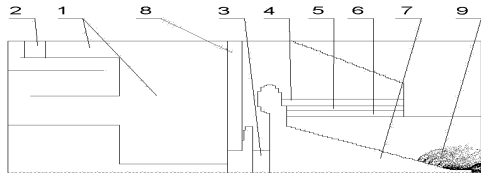
transformer oil.

Fig. 2. Calculated geometry of the STRAUS-2 accelerator.

At the initial time moment (t=0) DSFL the high-voltage electrode of which is charged to the voltage of 630 kV, discharging starts through the annular switch. The distribution of fields at such a switching represents a superposition of wave spreading from separate spark channels. The consideration of all fields in this case calls for the solution of a three-dimensional task. In the calculated model of the given paper the multi-channel switch is replaced by a circular axially symmetric switch, that provides the accuracy sufficient for practical calculations. The fact that the switch is a multi-channel one is taken into account through the introduction of the inductance constant by time to the calculated scheme [9, 10].
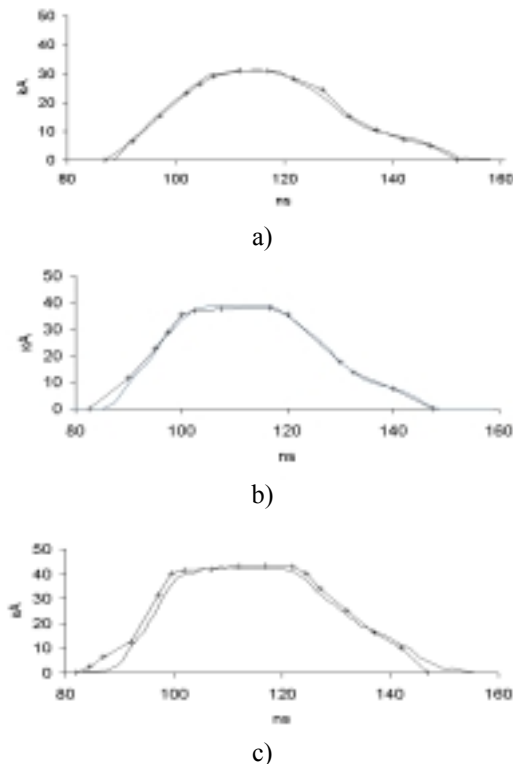

a)

b)

c)

Fig. 3. Calculated and experimental (+) current pulses in the diode at electrolytic loading resistance of 29 (a), 100 (b), 196 Ohm (c).

When the pre-pulse switch is turned on there arise no waves that are non-uniform in azimuth direction, thus, the given switch falls into the calculated scheme and is described by the inductance.

The full current in the area occupied by the switch is calculated by the following formula:

$$I = \frac{1}{L}\int_0^t U d\tau ,$$

where U – voltage between the switch electrodes, L – its inductance. Of the calculated I value there is defined the current density in the area occupied by the switch that is presented in the equation for the calculation of electric fields (6-8).

When calculating the STRAUS-2 accelerator the inductance of the annular switch turned out to be equal to 2.7 nH, the pre-pulse switch – to 70 nH, the cathode face diameter – to 30 mm and cathode-anode gap – to 50 mm.

The calculations of one of the modes of STRAUS-2 accelerator operation for the value of electrolyte resistance equal to 29, 100, 196 Ohm were performed (Fig. 3). The calculated current pulse parameters of the diodes were compared to the experimentally measured current pulses. It is evident from the comparison that the difference between the amplitude-time parameters of the calculated pulse and the parameters of the experimental pulse does not exceed 10% what lies within the limits of measurement accuracy.

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