ELECTROMAGNETIC OSCILLATIONS IN PERIODIC MEDIUMS AND WAVEGUIDES OUTSIDE THE PASSBAND

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INTRODUCTION

It is known that in boundless periodic mediums and waveguides there are two different in basis electromagnetic eigen oscillations supported by medium (waveguide) without external currents and charges [1]. In the certain frequency intervals (in passbands) the electromagnetic oscillations represent wave process which carry a constant energy (in the case of absence of damping) in direct or opposite direction. Between passbands the electromagnetic oscillations have a structure that is distinct from the previous case. In these frequency intervals electromagnetic oscillations transfer no energy in the direction of periodicity and have decreasing (increasing) dependence on the coordinate. These frequency intervals are called forbidden zones. Results of our investigations of the structure of electromagnetic oscillations in these intervals are represented. Such oscillations are similar to standing waves in the structures bounded from two sides that are the superposition of two travelling waves which move in opposite directions. Generally, outside passbands there are two kinds of eigen-oscillations: one decreases and another increases from the point of excitation. For semi-bounded mediums the increasing oscillations are not excited, and for waveguides, bounded from two-side, two oscillations are always excited, but the amplitude of the increased one, as a rule, is small.

Our results have shown that in the case of forbidden zone in the systems bounded from two sides there are situations when excited are mainly oscillations only of one type - being increased or decreased.

1. CAVITY CHAIN

The interest to a research of non-propagating oscillations in periodic systems was caused by our study of performances of injector systems on the basis of the standing waves. Such injectors form high -quality beams and accelerate them up to enough large energies [2, 3]. In the standing -wave injector systems the klystrom bunching mechanism is combined with acceleration. For this purpose it is necessary to create a special field distribution in which amplitudes increase from the initial injection point and there must be sufficiently long gaps without fields. Such distribution can be easily created by a selection of cavity coupling coefficients. Indeed, if the cavity chain can be described by equations

$$[\omega_n^2(1+\alpha_n)-\omega^2]\ \ A_n=\omega_n^2(\beta_{n-1}A_{n-1}+\beta_n A_{n+1}) \quad (1.1)$$

and such conditions are fulfilled

$$\omega_n^2(1+\alpha_n)=\omega_g^2, \quad (1.2)$$

then the field amplitudes A_n at this frequency $\omega=\omega_g$ is related by such a correlation

$$A_{n+1}=-\beta_{n-1}/\beta_n A_{n-1}. \quad (1.3)$$

If $|\beta_{n-1}/\beta_n|>1$ ($|\beta_{n-1}/\beta_n|<1$), field amplitudes are increasing (decreasing). Such question can be asked: what physical reasons are responsible for so nontrivial amplitude distribution?

To give answer on this question we shall consider the electrodynamic characteristics of a simple chain of cylindrical cavities with coupling through small cylindrical holes in separating cavity walls with zero thickness. We consider the case when the cavity geometric sizes and the hole radii are altered periodically.

We denote by A_n amplitudes of E_{010} oscillations in the cavities with length d_1 and radius b_1, and by B_n amplitudes in the cavities with length d_2 and radius b_2. We denote the hole radii by a_1 and a_2.

The set of equations for amplitudes has the form

$$\begin{aligned}&[\omega_1^2(1+\alpha_1)-\omega^2]A_n=\omega_1^2 B_n\beta_1+\omega_1^2 B_{n-1}\beta_2\ ,\\&[\omega_2^2(1+\alpha_2)-\omega^2]B_n=\omega_2^2 A_n\beta_1+\omega_2^2 A_{n+1}\beta_2\ ,\end{aligned} \quad (1.4)$$

where $\beta_{1,2}=\gamma\dfrac{a_{2,1}^3}{b_1 b_2\sqrt{d_1 d_2}}$, $\alpha_{1,2}=\gamma\dfrac{a_1^3+a_2^3}{b_{1,2}^2 d_{1,2}}$,

$\gamma=2/[3\pi J_1^2(\lambda_1)]$, $\omega_{1,2}=c\,\lambda_{01}/b_{1,2}$.

Since the system under consideration is periodic, the amplitudes fulfil such conditions

$$A_{n+1}=\rho A_n\ \ ;\ \ B_{n+1}=\rho B_n \quad (1.5)$$

It is easy to show, that at fixed frequency equations (1.4) have nonzero solutions for two values of the parameter ρ:

$$\rho_{1,2}=Q\pm\sqrt{Q^2-1}, \quad (1.6)$$

where Q is determined by the expression

$$Q=\frac{[\omega_1^2(1+\alpha_1)^2-\omega^2][\omega_2^2(1+\alpha_2)^2-\omega^2]}{2\omega_1^2\omega_2^2\beta_1^2\Delta}-\frac{1+\Delta^2}{2\Delta},$$

$$\Delta=\beta_2/\beta_1.$$

If $|\rho|=1$ ($|Q|<1$), we have wave process, otherwise ($|Q|>1$) - non-propagation. Analysis of dependence of Q on ω shows that at $0<\omega<\omega_{c,1}$ - $Q>1$, $\omega_{c,1}<\omega<\omega_{c,2}$ - $-1<Q<1$, $\omega_{c,2}<\omega<\omega_{c,3}$ - $Q<-1$, $\omega_{c,3}<\omega<\omega_{c,4}$ - $-1<Q<1$, $\omega>\omega_{c,4}$ - $Q>1$.

Thus, byperiodical cavity chain has two passbands separated by a forbidden zone. Q reaches the minimum value at frequency

$$\omega^2=\omega_*^2=\frac{1}{2}\left[\omega_1^2(1+2\alpha_1)+\omega_2^2(1+2\alpha_2)\right] \quad (1.7)$$

$$Q(\omega_*)=-f-(1+\Delta^2)/(2\Delta) \quad (1.8)$$

where

$$f=[\omega_1^2(1+2\alpha_1)-\omega_2^2(1+2\alpha_2)]^2/(2\omega_1^2\omega_2^2\beta_1^2\Delta).$$

For $a_1=a_2$ and fulfillment of the condition (1.2), the intermediate forbidden zone vanishes ($Q(\omega_*)=-1$) and so- called structure compensation happens [4], but in this case $\beta_1=\beta_2$ and creating the increasing (decreasing) distribution becomes impossible. In order to make values of amplitudes in intermediate cavities equal zero, it is necessary to create the standing wave mode [4]. If $a_1 \neq a_2$, then $\beta_1 \neq \beta_2$ and the intermediate forbidden zone does not disappear under any circumstances ($Q(\omega_*) < -1$), and condition (1.2) is reduced to the demand that the working frequency be equal to the mean frequency of the forbidden zone $\omega_g = \omega_*$. Thus, the creation of the increasing (decreasing) distribution requires working in the forbidden zone.

Let us consider in more details structure of such oscillations. It follows from the system (1.4), that when the cavity frequencies (including the coupling shifts) tend to the frequency that corresponds to the middle of the forbidden zone $\omega_1^2(1+\alpha_1) \to \omega_2^2(1+\alpha_2) \to \omega_*^2$ and $\rho = \rho_1$ the ratio B_n/A_n at $\omega = \omega_*$ tends to infinity, that is possible only in one case: $A_n \to 0$. When $\rho = \rho_2$ the ratio B_n/A_n tends to zero, i.e. $B_n \to 0$.

So, for two fundamental solutions in the middle of the forbidden zone the field structure has an interesting feature, both for the increasing solution and for decreasing one. There is a sequence of cavities (located through one) in which amplitudes are equal to zero. And, if for an increasing solution such cavities adjoin at the left to diaphragms with large holes, then for the case of decreasing solution they do to diaphragms with small holes.

Such a character of distribution of fields of non-propagating electromagnetic waves allows to create in the systems bounded from two sides the field distribution that corresponds to the field distribution of one from two fundamental solutions of a boundless case. The possibility follows from the circumstance, that the general solution of linear homogeneous equations is equal to the sum of partial solutions with unknown constants, which are determined from boundary conditions. Thus, by studying properties of fundamental solutions in a boundless system, we always know about possible distribution of a field in the bounded structure.

If we cut out a finite part of the biperiodical waveguide, ending it by a cavity which at the left has a diaphragm with a large hole, and makes some frequency shift in this cavity, we shall create a condition for supporting a field distribution that is appropriate to a decreasing fundamental solution. As was shown above, for such a solution the amplitudes of a field in cavities adjoin the diaphragms with large holes equal to zero, and for an increasing solution in cavities they adjoin the diaphragms with small holes equal to zero. Therefore in our case the amplitude of an increasing solution will be equal to zero. If we end a system by a cavity, which at the left side has a diaphragm with a small hole, we shall create a condition for supporting a field distribution that is appropriate to an increasing fundamental solution.

Thus, selecting in appropriate way boundary conditions, we can create in the bounded electrodynamic system the distribution of amplitudes that is appropriate only to one fundamental solution. It must be noted that it is possible only in the forbidden zone and only in case of a vanishing amplitudes in certain number of resonators. Within the passband these requests are defaulted.

On the one hand, such electromagnetic oscillations of the bounded volumes are similar to well-known eigen oscillations of resonators, which are obtained by partitioning a smooth or periodic waveguide by conducting plane walls with a frequency that lays in a passband of the waveguide, as they also do not transfer energy. On the other hand, the process of filling such volumes with electromagnetic energy is much different, as the non-propagating wave is excited.

2. MULTI-LAYER DIELECTRIC

Let us consider properties of eigen electromagnetic oscillations in a multi-layer dielectric, which represents periodically repeating along the axis z a set of laminae with thickness d_1 and d_2 and with permittivity ε_1 end ε_2. In transversal directions (x, y) the laminae are not bounded. We consider the elementary type of electromagnetic oscillations, in which the field strength does not depend on transversal coordinates. We assume the time dependence be $e^{-i\omega t}$. For this case the electromagnetic oscillations are described only by two components E_x and H_y. Dependence of these components from the longitudinal coordinate z can be found from the Maxwell equations and for two arbitrary selected adjoining laminae (i=1,2), from which the numbering periods (s = 0) are started, field components can be written as:

$$E_{x,i}^{(0)} = E_i^{(+)} e^{ik_{i,z}z} + E_i^{(-)} e^{-ik_{i,z}z}, \qquad (2.1)$$

$$H_{y,i}^{(0)} = \frac{\sqrt{\varepsilon_i}}{Z_0}\left[E_i^{(+)} e^{ik_{z,i}z} - E_i^{(-)} e^{-ik_{i,z}z}\right], \qquad (2.2)$$

where $k_{i,z} = \omega\sqrt{\varepsilon_i}/c$, and $E_i^{(\pm)}$ are constants.

As the system under consideration is periodic along the axis z with a period $D=d_1+d_2$, field components within the period with a number s will be determined by the expression:

$$E_{x,i}^{(s)} = \rho^s\left[E_i^{(+)} e^{ik_{i,z}(z-sD)} + E_i^{(-)} e^{-ik_{i,z}(z-sD)}\right], \qquad (2.3)$$

where ρ is some complex number. Similarly, one can write the expression for $H_{y,i}^{(s)}$. It is easy to show that at a fixed frequency the boundary conditions for field components are fulfilled only for two values of the parameter ρ [5]:

$$\rho_{1,2} = Q \pm \sqrt{Q^2 - 1}, \qquad (2.4)$$

where Q is determined by expression

$$Q = \frac{\alpha_1^2\cos(k_{1,z}d_1 + k_{2,z}d_2) - \alpha_2^2\cos(k_{1,z}d_1 - k_{2,z}d_2)}{\alpha_1^2 - \alpha_2^2},$$

$$\alpha_1 = (1 + \sqrt{\varepsilon_2/\varepsilon_1})/2, \quad \alpha_2 = (1 - \sqrt{\varepsilon_2/\varepsilon_1})/2.$$

Fig.1 shows the dependence of Q on the dimensionless frequency $\Omega = \omega D/(2\pi c) = D/\lambda$ for the case when $d_1=d_2=D/2$. One can see that in certain frequency intervals $|Q| > 1$, i.e. there are forbidden zones. Let us consider a structure of an electrical field in the first forbidden zone. Dependencies of the modulus of E_x on the longitudinal coordinate $\xi = z / D$ within one period (Ω =0.3, the middle of the first forbidden zone, $dQ / d\Omega \approx 0$) are shown in Fig.2. For this case Q=-1.2333 and ρ_1 =-0.5278, ρ_2 =-1.8948.

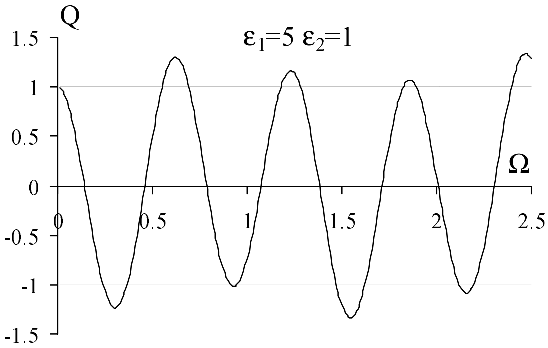

Fig.1.

From Fig.2 it follows, that both non-propagating eigen oscillations of multi-layer dielectric have an interesting feature - in certain planes perpendicular to axis z- the tangential component of an electrical field equals zero. And for two fundamental solutions these planes do not coincide. Putting in these places metal plates, we can effectively control the field distribution.

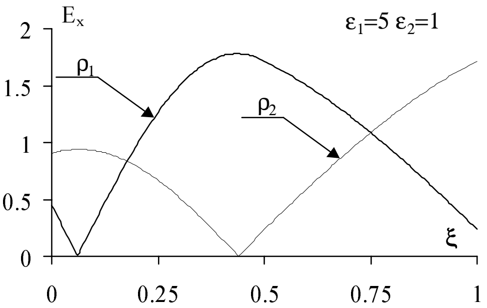

Fig.2

As was already mentioned above, a general solution of our electrodynamic problem is represented as the sum of two fundamental (partial) solutions $E(z) = C_1 E_1(z) + C_2 E_2(z)$. If the metal plate is located at $z = z_*$, such condition must be fulfilled

$$C_1 E_1(z_*) + C_2 E_2(z_*) = 0 \ , \qquad (2.5)$$

from which it follows that if $E_1(z_*) = 0$ ($E_2(z_*) = 0$), then C_2=0 (C_1=0). Thus, for definite z_*, we can create the field distribution that appropriate only to one fundamental solution, i.e. either pure increasing or decreasing one.

Using this circumstance, we can, at first, essentially change performances of waves reflected from a finite number of dielectric layers lying on the metal plate. Secondly, one can create dielectric resonance systems which have oscillations with increasing (decreasing) field distributions along the coordinate z.

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