STUDY OF ULTRARELATIVISTIC BEAM EMITTANCE

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We estimated a minimum radius of a beam under given normalized emittance and a minimum emittance of proton and electron beams. We obtained the simple expression for a minimum radius of a beam $x_{\min} = E_n/1$ rad and the value of the minimum for normalized emittance for proton and electron beams $E_{n,1} \sim 0.01$ cm·mrad and $E_{n,2} \sim (0.01 \div 1)$ cm·mrad respectively.

PACS number: 29.27.Fh

We considered charge particle beams with acceleration duration τ in the linac which is more than a typical relaxation time τ_0 (the equilibrium state due to close collision). In this case the beam can be characterized by a certain value of temperature T_0. Our estimation for most of proton beams in resonance linacs $\tau_0 = \tau_{pp}$ ~1 s. Practically this means the existence of a stationary beam temperature at the next stage of acceleration – circular accelerators with a minimum output energy ~ 10 GeV. The stationary electron beam temperature takes place significantly earlier. We considered properties of such charge particle beams. Expression for the total particle velocity β_Σ is

$$\beta^2{}_\Sigma = \beta^2 + \beta^2{}_{\perp m} - \beta^2\ \beta^2{}_{\perp m} \qquad (1)$$

where β_Σ, β are the normalized total and longitudinal velocities in a laboratory coordinate system respectively, $\beta_{\perp m}$ is the transversal normalized velocity in a moving system of coordinates. From (1) resulting is the expression for the transversal normalized velocity in the laboratory system of coordinates

$$\beta_\perp = \beta_{\perp m}/\gamma \qquad (2)$$

where $\gamma = 1/\sqrt{1-\beta^2}$.

Then the maximal value of the transversal velocity for given γ is

$$\beta_{\perp\max} = 1/\gamma \qquad (3)$$

For ultrarelativistic beams we have the expression $1-\beta \ll 1$, then the maximum value of divergence $x' = dx/dz = \beta_{\perp\max}/\beta$ practically coincides with the value $\beta_{\perp\max}$ (x, z are transversal and longitudinal coordinates respectively).

Basis for any emittance definition [1, 2] is the integral invariant $\int dx dp_x$ = const, where x, p_x are the transversal coordinate and momentum of a particle. If the emittance shape has a canonical form of ellipse, i.e. its semiaxis coincides with the coordinate axis, then taking into account (2) it is easy to determine the expression

$$\left(xp_x\right)_m = \left(xp_x\right)_l \qquad (4)$$

The indexes l and m are applied to laboratory and moving systems respectively. From expression (4) it follows that the integral invariant $\int dx dp_x$ is the relativistic invariant too.

If the shape of the normalized emiittance $E_н$ is as an ellipse of a canonical form, then

$$E_н = xx'\beta\gamma = x\beta_\perp\gamma \qquad (5)$$

From expression (5) taking into account (3) it follows that the minimum radius of ultrarelativistic beams

$$x_{\min} = E_н/1\text{rad}. \qquad (6)$$

Thus, the minimum radius of ultrarelativistic beams (6) takes a place under acceleration of ultrarelativistic beams with a constant value of emittance $E_н$, an effort to produce the beam with the radius $x < x_{\min}$ will cause losses of charge particles.

Normalized emittances of ultrarelativistic proton (γ~80) [3] and electron (γ~4000) [4] beams in resonance accelerators are $E_н$~100 cm·mrad, then from (6) results that $x_{\min} \sim 0.1$ cm. The output transversal normalized velocities of the electron beam for the linac LU-2 [4] are $x' \simeq \beta_\perp \le 10^{-4}$, the value $\beta_{\perp\max} = 2.5\cdot10^{-4}$ for the output beam energy W = 2 GeV.

Expression (2-6) are valid under low energies too, it enables to estimate the minimum emiittance from the gun. From expression (6) results that the estimation of a minimum emittance as far as possible is connected with the minimum of beam radius as far as possible. The minimum of beam radius can be determined using the quantum mechanics limitation for a transversal dimension of the wave packet [5]. For a proton beam this minimum is $x_{\perp\min} \sim 10^{-5}$ cm, for an electron beam it is $x_{\perp\min} \sim (10^{-3} \div 10^{-5})$ cm. Then from (6) we can estimate as far as possible the minimum of a normalized emittance. For a proton beam from the gun it is $E_{н.\min} \sim 0.01$ cm·mrad and for electron beam it is $E_{н.\min} \sim (0.01 \div 1)$ cm·mrad.

Most of real emittance values for proton and electron beams from the gun are higher than the minimum one by an order of magnitude at least. Thus, we estimated the minimum of beam radius as far as possible (6) for the given value of a normalized emittance and, next, the minimum emittance as far as possible for proton and electron beams for the given minimum of beam radius as far as possible.

REFERENCES

1. I.M.Kapchinsky. *Charge particle dynamics in linear resonance accelerators*. Moscow: Atomizdat, 1966 (in Russian).
2. V.M.Khoruzhiy et al. About multi-charge ion beam emittance // *Voprosy Atomnoj Nauki i Tekhniki. Ser. Tehnika fizicheskogo ehksperimenta* (19). 1984, v. 2, p. 58-59 (in Russian).
3. E.G.Komar. *Basis of acceleration technique*. Moscow: Atomizdat, 1975, 322 p. (in Russian)
4. V.I.Artemov, G.K.Demyanenko, N.A.Kovalenko, F.A.Peev. Development of focusing system for LU-2 linac // *Voprosy Atomnoj Nauki i Tekhniki, seriya: Fizika Vysokikh Energij* (1). 1972, v. 1, p. 64. (in Russian)
5. A.M.Shenderovich. About dynamics and formation of beams in linacs // *Zhournal Tekhnicheskoj Fiziki*. 1983, v. 53, # 6, p. 1078-1081 (in Russian).