CODE PHASCOL FOR COMPUTATION OF SPACE CHARGE EFFECTS IN THE CYCLOTRONS AND SYNCHROCYCLOTRONS

L.M. Onischenko, E.V. Samsonov
Joint Institute for Nuclear Research (JINR), Dubna, Russia

Development of the code is connected with the project of external injection into the JINR Phasotron. General description of the code PHASCOL intended for particle dynamic calculation that takes into account Coulomb interaction between cycling particles is given. A method of large particles is used in the code. Visual graphical possibilities of the Microsoft FPS 4.0 are applied for on-line representations of the main results of computations. Contemporary PC makes it possible to carry out in the course of 24 hrs the calculations of interaction of 2000 particles for 100 turns. Some first results of computation regarding to the JINR Phasotron with energy 680 MeV and four-sector cyclotron-injector with energy 5 MeV and beam currents up to 10 mA are presented.
PACS numbers: 29.20.Bd, 83.10.Rs

1 GENERAL DESCRIPTION OF CODE PHASCOL

The code PHASCOL is intended for a particle dynamic computation taking into account the space charge effects (SCE). For numerical modeling of SCE a beam is subdivided into a series of bunches. Each bunch contains a set of large particles. To correct calculation of an electric field inside the bunch the overall charge of particles corresponds to the beam current that is modeled. The following full differential equations [1] of particle motion written in cylindrical coordinate system are used in PHASCOL:

$$\ddot{r} = r\dot{\varphi}^2 + \frac{qc^2}{E}[\varepsilon_r - r\dot{\varphi}B_z + \dot{z}B_\varphi - \frac{\dot{r}}{c^2}(\dot{r}\varepsilon_r + r\dot{\varphi}\varepsilon_\varphi + \dot{z}\varepsilon_z)]$$

$$\ddot{z} = \frac{qc^2}{E}[\varepsilon_z + r\dot{\varphi}B_r - \dot{r}B_\varphi - \frac{\dot{z}}{c^2}(\dot{r}\varepsilon_r + r\dot{\varphi}\varepsilon_\varphi + \dot{z}\varepsilon_z)]$$

$$\ddot{\varphi} = -\frac{2\dot{r}\dot{\varphi}}{r} + \frac{qc^2}{rE}[\varepsilon_\varphi + \dot{r}B_z - \dot{z}B_r - \frac{r\dot{\varphi}}{c^2}(\dot{r}\varepsilon_r + r\dot{\varphi}\varepsilon_\varphi + \dot{z}\varepsilon_z)]$$

where (r, φ, z) – particle coordinates, (B_r, B_φ, B_z), $(\varepsilon_r, \varepsilon_\varphi, \varepsilon_z)$ – components of the magnetic and electric fields, q - particle charge, c – velocity of light, E – full particle energy, dot means a differentiation respect to time. The Runge-Cutter method of 4-th order is used for numerical integration of the equations. All computations are fulfilled in double precision mode.

The magnetic field is entered making use the radial dependencies of Furrier's harmonics in a median plane of accelerator. Outside the median plane the magnetic field components are computed in accordance to the Maxwell's equations. A magnetic field created by the beam current is not considered by the code.

The electric field is represented as a sum of an acceleration system field and a proper beam field:

$$\varepsilon_{r,\varphi,z} = \varepsilon^{RF}_{\varphi,z} + \varepsilon^{SC}_{r,\varphi,z}\,.$$

The acceleration system field is written as:

$$\varepsilon^{RF}_{\varphi,z} = \varepsilon^{\max}_{\varphi,z}\cos(2\pi\int_0^t f(t)dt + \psi)\,,$$

where: $f(t)$ – frequency program of the synchrocyclotron (it is constant for the cyclotron), ψ – particle starting phase relatively accelerating field. The analytical description [2] is adopted for calculation of the amplitudes $\varepsilon^{\max}_{\varphi}$, $\varepsilon^{\max}_{z}$. To compute the proper field of bunch, on each step of integration the following expressions are used:

$$\varepsilon(i)^{SC}_r = \frac{q}{4\pi\varepsilon_0}[\cos\varphi_i\sum_{k=1}^{k=N}(x_i - x_k)/d^3_{ik} + \sin\varphi_i\sum_{k=1}^{k=N}(y_i - y_k)/d^3_{ik}\ ],$$

$$\varepsilon(i)^{SC}_\varphi = \frac{q}{4\pi\varepsilon_0}[\cos\varphi_i\sum_{k=1}^{k=N}(y_i - y_k)/d^3_{ik} - \sin\varphi_i\sum_{k=1}^{k=N}(x_i - x_k)/d^3_{ik}\ ],$$

$$\varepsilon(i)^{SC}_z = \frac{q}{4\pi\varepsilon_0}\sum_{k=1}^{k=N}(z_i - z_k)/d^3_{ik}\,,$$

$$d_{ik} = ((x_i - x_k)^2 + (y_i - y_k)^2 + (z_i - z_k)^2)^{1/2}\,,$$

where i – the number of particle in the location of which is calculated electric field, $k \neq i$ – the numbers of other particles of the bunch, d_{ik} - distance between i-th and k–th particles, (x, y, z) - Cartesian coordinates of the particles, ε_0=8.85·10^{-12} F/m.

If the distance between the particles is less than 0.01 mm, then interaction between them is ignored in order to avoid noise effects. For the same aim of avoiding noise effects, the bunch field distribution is recomputed on each step of integration. Work experience with the code showed that out of the field of the accelerating system the average energy of bunch remains constant with the accuracy ±10 eV.

On each step of integration a particle gets or loses an energy in accordance to the sign of the expression:

$$\Delta w = q(\dot{r}\varepsilon_r + r\dot{\varphi}\varepsilon_\varphi + \dot{z}\varepsilon_z)\Delta t\,,$$

where Δt – is time step of integration.

In PHASCOL is a possibility of adding the specific number of injected bunches, which follow through the assigned time intervals. This option of the code permits to model an external injection into JINR Phasotron.

For visual representation of the input data and the results of computation the graphical possibilities [3] of the Microsoft FPS 4.0 are applied. These graphs show a staring and final position of the particles on the transverse and longitudinal phase planes, the radial and axial particle trajectories on the phase planes as well as versus time and azimuth angle. A signal of differential probe located on arbitrary azimuth and having definite radial size is also computed and shown as a graph versus radius.

2 TWO EXAMPLES OF SCE SIMULATION

Computer simulation of the beam dynamics was carried out for the external injection (Fig. 1) into the Phasotron [4] of the preliminary bunched 5 MeV beam as well as inside an injector. It was proposed that a 4-sector cyclotron CI-5 be used as the injector for the Phasotron. In order to get an average beam current of about 50 μA in the Phasotron the injected beam must be of order 10 mA during a capture time duration ~45 μs.

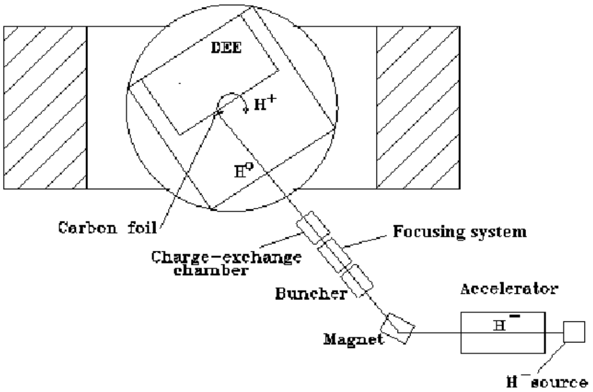

Fig. 1. Phasotron external injection scheme.

2.1 Simulation of sce in 4 sector cyclotron-injector

Some important parameters of the cyclotron-injector CI-5 are shown in Table 1.

Table 1.

Type of accelerated particle		H^-
Initial energy	(MeV)	0.5
Final energy	(MeV)	5.0
Average magnetic field	(T)	0.43
Betatron frequencies:	ν_r	1.1
	ν_z	0.8-0.9
Radius of injection	(cm)	27.0
Radius of extraction	(cm)	75.0
Emittances on injection	(π mm·mrad)	50.0
Phase width of the bunch	(°RF)	20
Orbital frequency	(MHz)	6.7965
Harmonic number		8
Number of acceleration gaps		4
Accelerating voltage	(kV)	115-200

Simulation of particle dynamics was done for a single bunch accelerated during 7 turns needed to reach the final energy 5 MeV. Preliminary a set of particles had been randomly distributed inside this bunch at starting position. From 500 to 5000 particles were used in computations. It was observed, if the number of particles was larger than 2000, then final results (such as emittances or transverse sizes) become fixed. Therefore, to decrease time consuming, the computations were fulfilled mainly for 2000 particles.

Figures 2, 3 illustrate the results of computations done without space charge forces and taking into account them (I=10 mA) in horizontal and vertical planes. It is seen the SCE lead to essential increase of the bunch size in radial and axial dimensions. A free radial zone between 7-th and 6-th turns required for effective extraction decreases from 25 to 3 mm. At the same time the SCE induce an enlargement of axial beam size by approximately two times.

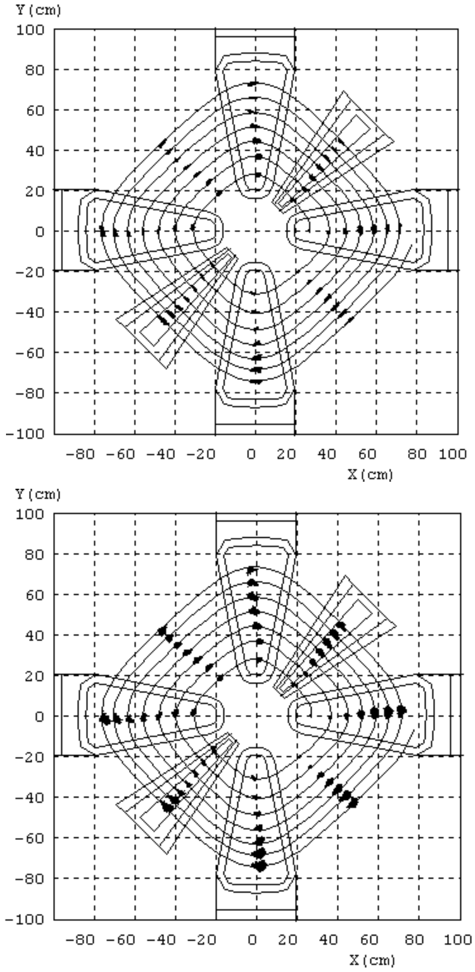

Fig. 2. Comparison of the bunch form during acceleration. Complete trajectory of one particle and position of all particles after each 45° are shown. Above – no SCE, below – with SCE (10 mA).

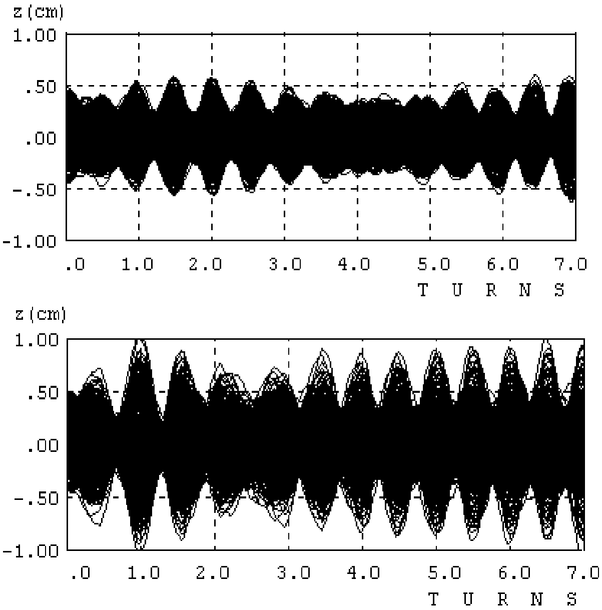

Fig. 3. Comparison of particle axial trajectories during acceleration in CI-5. Above – no SCE, below - SCE (10 mA).

2.2 Simulation of sce in the phasotron

It is supposed that the beam delivered from the cyclotron after additional bunching and neutralization ($H^- \rightarrow H^0$) is injected at the central region of the Phasotron. Carbon foil will be used in order to get proton beam ($H^0 \rightarrow p$). Some parameters of the Phasotron central region for the scheme of external injection are given in Table 2.

Table 2.

Type of accelerated particle		p
Initial energy	(MeV)	5.0
Average magnetic field	(T)	1.2
Betatron frequencies:	ν_r	1.01
	ν_z	0.12
Radius of injection	(cm)	27.0
Phase width of the bunch	(°RF)	18
Harmonic number		1
Number of acceleration gaps		2
Accelerating voltage	(kV)	37

There are three important distinctions between the initial parameters on SCE simulation in CI-5 and Phasotron. First, a longitudinal size of the bunch injected in the Phasotron is approximately two times greater than this size at the cyclotron extraction region. Second, a frequency of axial free oscillations in the central region of Phasotron (0.12) is appreciably smaller than in the cyclotron (0.9). Third, an energy gain per turn in the Phasotron is so small that at least 20 successive bunches overlap each other at azimuth of injection. So, the accumulation of the space charge of beam occurs in the injection domain. Having in mind last occurrence, 20 bunches were used in computations for accurate simulation of SCE in the Phasotron. The particles were used with the initial amplitudes of transverse oscillations not more than 15 mm.

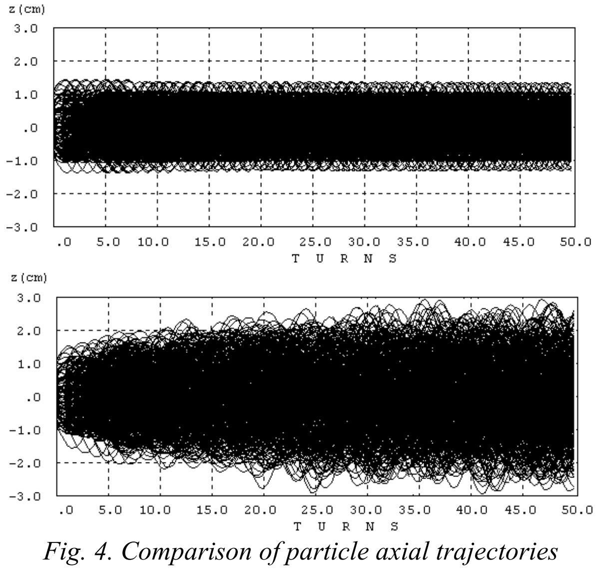

Fig. 4. Comparison of particle axial trajectories during acceleration in the Phasotron. Above – no SCE, below – with SCE (10 mA).

The results of computation of the axial particle motion both without taking into account SCE and considering SCE for the current 10 mA are compared in Fig. 4. In these calculations was simulated the sequential injection of 20 bunches, each of which contained 100 particles. Then, up to 50 revolutions was calculated the motion of one bunch, which consists of 2000 particles. It is evident that the space charge leads to an increase in the axial size of beam from 28 to approximately 50 mm.

Calculations did not show distinct space-charge effect on the amplitude of particles radial oscillations. At the same time, the SCE leads to noticeable changes of the bunch form and energy spread. Fig. 5 shows as it changes on the plane an azimuth-radius the form of the bunch, which consists of 2000 particles, under the action of the forces of space charge. It is evident that on 50 turn the deformation of bunch acquires the characteristic form, which give calculations [5] after 5 turns in PSI Injector 2.

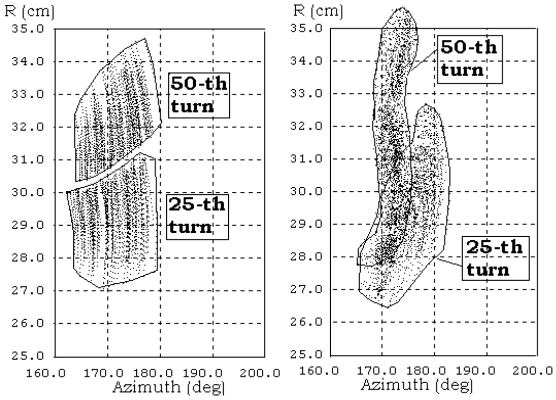

Fig. 5. Position of particles on the plane azimuth-radius during 25-th and 50-th turns. To the left – without taking into account space charge, to the right -taking into account it.

CONCLUSIONS

Code PASCOL gives important information regarding the beam behaviour influenced by the space charge forces. No noise effects were detected if number of large particles was of order several thousands. It is desirable to fulfill analogous calculations by means of other codes and to compare the obtained results.

REFERENCES

1. V.V.Kolga. Mathematical Simulation on Computers of Dynamic Processes in the Accelerators // *Proc. International School of the Young Scientists*, Minsk, 1979, p. 300, (in Russian).
2. N.Hazewindus et al. The Magnetic Analogue Method as Used in the Study of a Cyclotron Central Region // *NIM*. 1974, v. 118, p. 125.
3. O.V.Bartenev. *Visual Fortran, New Possibilities*. Moscow: Dialog-MFEI, 1999, (in Russian).
4. L.M.Onischenko, E.V.Samsonov. External Injection into JINR Phasotron. Computer Simulation II // *Proc. of International Conference on Cyclotrons and Their Applications*, East Lansing, 2001, P4-04.
5. S.Adam. Space Charge Effects in Cyclotrons-From Simulations to Insights // *Proc. 14 Int. Cycl. Conf.*, Cape Town, 1995, p. 446-449.