QUADRUPOLE LENS WITH THE CONTROLLED SEXTUPOLE COMPONENT

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In the paper the quadrupole lens with a controlled sextupole component is considered. The results of magnetic field modeling for various modes of operations of a lens are shown. The required field is realized with the help of the special pole shape and by the additional excitation windings. The lens described is supposed to be used in the compact cyclic accelerators, sources of synchrotron and X-ray radiation.
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1 INTRODUCTION

There are many designs and operating installations, in which the quadrupole lenses satisfy additional functions. So, for example, in compact X-ray radiant synchrotron (ISI-800 [1], N-100 [2]) it is planed to use quadrupole lenses with a sextupole component. In the process, the range of the sextupole component variation for all modes of operations, as a rule, is either positive or negative concerning the quadrupole component. Usually two methods of realization of a sextupole component are used. One of them is optimization of a pole shape, the second is location inside a lens and on yoke, an additional current coils [3]. By fitting the coil currents it is possible to achieve a required sextupole component. Combining an invariable part of a sextupole component (by pole shape) with controllable one (by currents) is probably essential to increase the sextupole component magnitude.

2 MATHEMATICAL MODEL FOR CALCULATION OF THE POLE SHAPE

For calculation of a quadrupole lens with a sextupole component the following assumption is accepted:

- the field on a pole does not exceed 1T and therefore it is possible to use an approximation μ=∞;
- the pole shape is calculated without taking into account the edge fields and, therefore, lens can be described in the terms of potential functions and with usage of a complex plane.

As is known the complex potential of a quadrupole+sextupole field on a complex plane $\omega=x+iy$ looks like:

$$Z(\omega)=\frac{G_2}{2!}\omega^2+\frac{G_3}{3!}\omega^3 \qquad (1)$$

Constancy of the imaginary part of the complex potential (the scalar potential) sets equipotential lines. These lines can be obtained by a solution of equation (1) relative to ω. This solution has 3 branches corresponding to 3 poles with identical potentials:

$$W_1(z)=-\frac{G_2}{G_3}+G_2^2\chi^{-\frac{1}{3}}+\chi^{\frac{1}{3}}; \qquad (2)$$

$$W_2(z)=-\frac{G_2}{G_3}-\frac{1+i\sqrt{3}}{2}G_2^2\chi^{-\frac{1}{3}}-\frac{1-i\sqrt{3}}{2}\chi^{\frac{1}{3}}; \qquad (3)$$

$$W_3(z)=-\frac{G_2}{G_3}-\frac{1-i\sqrt{3}}{2}G_2^2\chi^{-\frac{1}{3}}-\frac{1+i\sqrt{3}}{2}\chi^{\frac{1}{3}}; \qquad (4)$$

where $\chi=\dfrac{G_2^2}{G_3\left(3G_3^2z-G_2^3+\sqrt{3}\sqrt{3G_3^4z^2-2G_2^3G_3^2z^2}\right)}$.

Expressions (2-4) are parametric representation of the pole shape for a quadrupole lens with a sextupole component. As a parameter in these expressions the abscissa of a complex plane $z=(t+is)$ is used. Having set particular values s=const, it is possible to obtain the shape of poles of a required multipole (see Fig. 1).

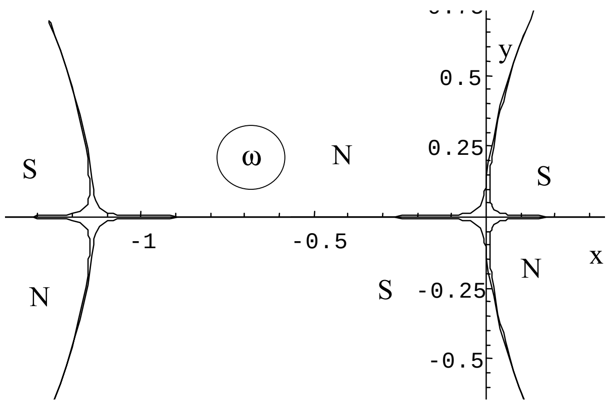

Fig. 1. The pole shapes of the "ideal" quadrupole+sextupole lens. Measurements are in meters. G_2=20T/m, G_3=35T/m^2.

From reasons of a symmetry it is obvious, that the multipole under consideration is symmetrical concerning an axis 0X. Both quadrupole crosses are symmetrical relative to an axis [*Const, Y*]. For us the practical meaning has only the area (0, 0). Therefore for description of the qudrupole+sextupole installation we shall terminate solutions (2) and (4) (see Fig. 2).

Expression (2) is formal maps a rectilinear band 0≤Im(z)≤H on a band, whose upper coast is formed by a curve L1 (see Fig. 2), obtained with the help of expression (2) with a set value of a scalar potential (s=Im(z)=H) on the upper coast of a band. The lower coast of a band is formed by a curve S and ray [0, ∞].

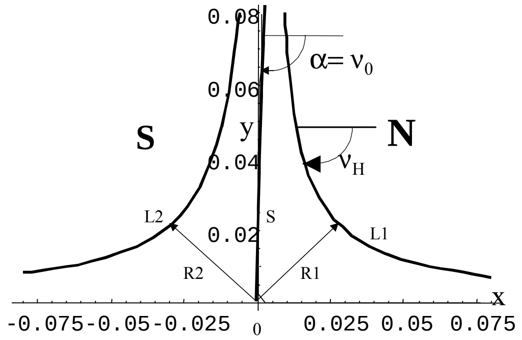

Fig. 2. The upper half of "ideal" multipole.

This can be obtained from expression (2) for a case of a scalar potential equal to zero (s=Im(z)=0). Similarly, expression (4) is the map of the rectilinear band 0≤Im(z)≤H on a band, whose upper coast is formed by a curve L2 (see Fig. 2), obtained with the help of expression (4) with a set value of a scalar potential (s=Im(z)=-H) on the upper coast of a band. The lower coast of a band is formed by a curve S and ray [-∞, 0], that is obtained from expression (4) for a case of a scalar potential equal to zero (s=Im(z)=0). The value of a scalar potential H can be found from a relation

$$R1 = \left|W_1(iH)\right|, \tag{5}$$

associating a point (0, iH) of a plane z with a point (Rx, Ry; |Rx, Ry | = R1) of a plane ω. The value R2 can be obtained from the expression:

$$R2 = \left|W_3(iH)\right|, \tag{6}$$

In practical manufacturing a lens with given values G_2, G_3 by formulas (2, 4) there are difficulties, connected with that the pole can not have an infinite extent. A problem where and how to break off a pole providing the practical realization of a lens, we shall solve with the help of the approach explained in operations [3, 4]. In frameworks of this approach the function being inverse to complex potential realizing the map of a rectilinear band on a polar band is searched. This function looks like:

$$\omega(z) = C\int_{z_0}^{z}\exp[G(z)]dz \tag{7}$$

where:

$$G(z) = \frac{1}{2H}\left\{\int_{-\infty}^{\infty} \nu_0(t)\left[\operatorname{cth}\frac{\pi(t-z)}{2H} - \operatorname{th}\frac{\pi t}{H}\right]dt - \int_{-\infty}^{\infty} \nu_H(t)\left[\operatorname{th}\frac{\pi(t-z)}{2H} - \operatorname{th}\frac{\pi t}{H}\right]dt\right\}.$$

Where the function $\nu_0(t)$ is an inclination of the lower coast of a polar band (see Fig. 2); the function $\nu_H(t)$ is an inclination of the upper coast of the polar band (see Fig. 2).

And it is possible to show that to implement a required field (1) in the 1-st quadrant of a lens (see Fig. 2), it is necessary, that the requirements were fulfilled:

$$\begin{aligned} \nu_0(t) &= \operatorname{Re}\left(i\ln\left(\frac{d(W_1(z))}{dz}\right)\right)\Bigg|_{\operatorname{Im}(z)=0}; \\ \nu_H(t) &= \operatorname{Re}\left(i\ln\left(\frac{d(W_1(z))}{dz}\right)\right)\Bigg|_{\operatorname{Im}(z)=H}. \end{aligned} \tag{8}$$

For the second quadrant these requirements look like:

$$\begin{aligned} \nu_0(t) &= \operatorname{Re}\left(i\ln\left(\frac{d(W_3(z))}{dz}\right)\right)\Bigg|_{\operatorname{Im}(z)=0}; \\ \nu_H(t) &= \operatorname{Re}\left(i\ln\left(\frac{d(W_3(z))}{dz}\right)\right)\Bigg|_{\operatorname{Im}(z)=-H}. \end{aligned} \tag{9}$$

Using the realizable pole which is not extending to infinity, the equalities (8, 9) can be realized only approximately in the area of a point (0, 0). In a real lens the inclination of a pole shape ($\nu_H(t)$) is set as follows:

$$\nu_H(t) = \begin{cases} \lambda, & t \in [-\infty, a_1]; \\ \sum_{0}^{K} q_i t^i & t \in [a_1, a_2]; \\ \theta, & t \in [-\infty, a_2]. \end{cases} \tag{10}$$

Such representation of the inclination of the pole shapes describes the pole with rectilinear outside inclinations and inside curvilinear segment between points a_1, a_2. Thus, having selected coefficients q_i so that the function inverse to complex potential ω(z) (7) be coincided with similar functions $W_1(z)$ (2) and W_2 (z) (4) in the corresponding quadrant, with a necessary exactitude it is possible to realize a quadrupole lens with a set sextupole component.

3 CALCULATION OF THE LENS

For calculation of the lens following parameters (for ISI-800 [2]) are required:

Quadrupole gradient G_2	20T/m.
Sextupole gradient G_3	0÷70T/m^2.
Aperture radius	0.035м.
Effective length	0.3м.

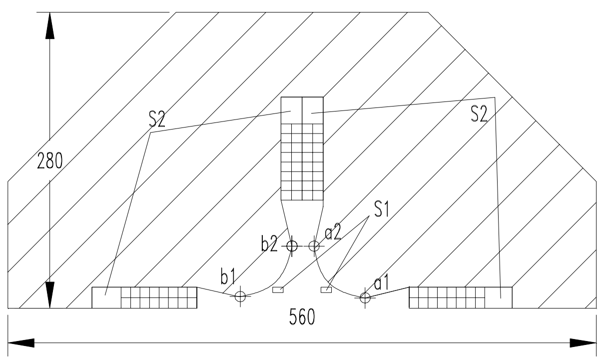

Fig. 3. The cross-section of the upper half of lens.

Portion of a sextupole gradient formed by the pole shape, will be defined by the magnitude 35T/m^2. The pole shape, defined coordinates of points of segments [a_1, a_2 and b_1, b_2] (see Fig. 3), was obtained by the method explained in the previous item for 2 quadrants. The outside inclination are selected identical and are equal to 14°. Compatibility of current windings S1, S2 (see. Fig. 3) allows to realize a sextupole gradient in the area ±40 T/m^2. The winding S1, located near to the pole gives the contribution to the lens field of shown in Fig. 4, curve S1.

The winding S2 gives the contribution to the lens field shown in Fig. 4, curve S2. From the figure it is senn, that the dipole component of coils has different signs, while the sextupole component has some signs. Switching in appropriate way these coils raise the sextupole component of the field in lens (see Fig. 4, curve S2+S1).

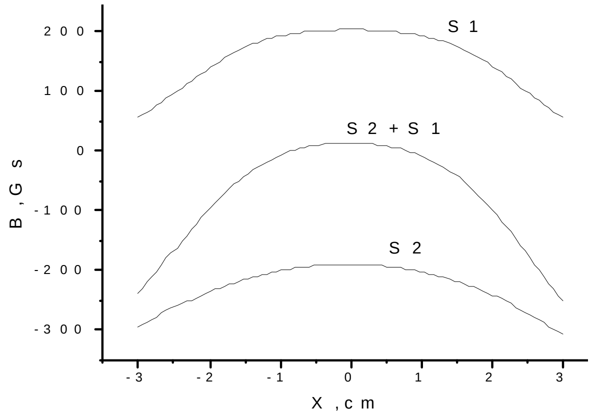

Fig. 4. Making sextupole component in a quadrupole lens.

The quality of a lens field is defined first of all by the value of the octopole component. In Table 1 listed are the values of the basic nonlinearities of the field (B=$\sum_{i=2} G_i z^i$) at various values of sextupole component. In table1 the currents exciting the winding Q, S1 and S2 (see Fig. 2) are listed also.

Table 1.

Parameter	$G_{3.}$=0 T/m^2	$G_{3.}$=35 T/m^2	$G_{3.}$=70 T/m^2
G_2, T/m	20	20	20
G_4, T/m^3	-110	-48	36
G_5, T/m^4	-24·10^4	0.24·10^4	50·10^4
G_6, T/m^5	-101·10^6	1.4·10^6	2.6·10^6
Cur. .Q,A	10000	10000	10000
Cur. S1,A	500	0	-500
Cur. S2,A	423	0	-423

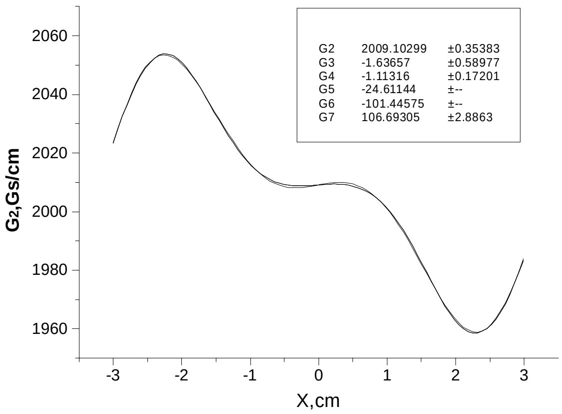

Fig. 5. Distribution of the gradient in lens in a mode of compensated sextupole component.

Fig. 5 shows the distribution of a gradient in a working area for a mode, when the sextupole component of fields is equal to zero.

In Table 2 the basic parameters of a lens are given.

Table 2. Parameters of quadrupole lens with sextupole component

Parameter	Value
Range of modification of the quadrupole gradient Gmin÷Gmax T/m	0÷20
Range of modification of the sextupole gradient G_3min÷G_3max T/m^2	0÷70
Working area mm×mm	±20×±9
Aperture radius, mm	35
The maximum power of the basic magnetizing coil, kw	6.5
4 parallel branches of cooling	
Overheating the water in the basic winding, °C	11
Current of the basic winding, A	600
Voltage drop, V.	11
Mass of the lens, kg	35+550

4 CONCLUSION

The lens with calculated parameters completely meets the requirements following from dynamics of particles in the installation ISI-800. The application of a quadrupole lens with a controllable sextupole component will allow to reduce the requirements to correcting devices of cyclic accelerators.

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