CHAOTIC DYNAMICS OF BEAM PARTICLE INTERACTING WITH STANDING WAVE FIELD OF SHORT CAVITY

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Chaotic dynamics of nonrelativistic beam particles interacting with a standing wave field of a short cavity under conditions closed to Cherenkov's resonance was studied. The criterion of development of stochastic beam particle movement instability in the standing wave field was obtained. It was shown that at small field amplitude being excited in the cavity with the beam dynamics of beam particles is not regular because of overlapping the parametric resonanses. This causes appearance of back particles, i.e., the particles leaving the cavity across input end and results in decreasing the electron efficiency because of coherently emitting bunch distortion.
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1 INTRODUCTION

The spectrum of oscillations excited by the beam moving in a slow-wave cavity may be considered as discrete if the linear increment $\gamma << \Delta\omega$, where $\Delta\omega$ is the distance between natural cavity frequencies. In the region where phase velocities closed to the beam velocity (Cherenkov's resonance)[1], $\Delta\omega \approx V_o \Delta k$, V_o is the initial beam velocity, $\Delta k \propto \pi / L$. This means that for the time of particle beam passing through the cavity $T = L / V_o$ the field amplitude does not change[2]: $\gamma T << \pi$ ("short" cavity of slow waves). Significant increase of the field amplitude in the cavity may be reached due to the effect of accumulation of the energy lost by incoming, in-sequence, beam particles. The high quality of the cavity is required.

The process of regular oscillation excitation in the "short" slow-wave cavity by the magnetized electron beam of a low density in the detuning region closed to Cherenkov's resonance qualitatively consists in the following. At first, the particles incoming into the cavity transmit a low part of their kinetic energy to the field and rapidly leave the latter. The field amplitude growths with the increment γ. In time the part of beam particles are trapped by the wave excited by the beam. The trapped particles being in the accelerating phase rapidly leave the cavity. The particle being in the moderated phase gather in bunches which "live" for a long time in the cavity, displacing along the phase of excited field they transmit their kinetic energy to the field and leave the cavity. The field growth is over when for the time of flight through the cavity the bunches formed because of interaction with the field can accomplish approximately the half phase oscillation:

$$\Omega_{ph} T \approx \pi , \ \Omega_{ph}^2 = |e| Ek / m .$$

Approximately fourth part of the phase oscillation corresponds to the maximum electron efficiency. These observations allow to estimate the maximum amplitude of the wave field excited in the cavity (exact theory see in [1]: $E_{\max} \propto \pi m V_o^2 / (n |e| L)$, where n is the mode number. The maximum energy losses are reached at $E_{\max}^{\eta} \propto \pi m V_o^2 / (4n |e| L)$.

The beam particles in the cavity move in the field of two slow waves – straight one propagating along the beam and back one propagating opposite to the beam. Their dynamics is stochastic if the Cherenkov's resonances corresponding to interaction between straight and opposite waves will be overlapped: $E_{ov} = m\omega_n^2 L / (4\pi |e| n)$, where ω_n is the frequency of excited oscillations. It is easy seen that for n ~ 1 $E_{\max}$ may be higher then E_{ov} [3]. In this case the beam particles move not regularly. This leads, firstly, to appearance of back particles which leave the cavity through the input end (they may occur in the region of the phase space with negative velocities, in particular they may be trapped by the back wave), secondly, to decrease of the efficiency of field interaction with the field of excited oscillations, because of total or particular distortion of coherently emitting bunches, thirdly, the amplitude of the beam excited field is defined by the not regular mechanism considered above, but by the chaotic dynamics of beam particles[4]. Chaotic dynamics does not influence on characteristics of the system if the time of split correlation is more then the time-of-flight.

The presented picture of the beam- cavity interaction is qualitative. Below, in particular, it is shown that the chaotic dynamics of beam particles is developed at smaller field amplitudes due to existing parametric resonances.

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[1] In the case of the plasma layer $\Delta\omega \approx V_o \Delta k$, where $\Delta k \propto \pi / L$, L is the cavity length. In the case when the cavity is filled with a dielectric: $\Delta\omega \approx V_o \Delta k$, $\Delta k \propto \zeta^2 \pi / L$, where $\zeta << 1$, $\zeta \propto L / a$, or $\zeta \propto a / L$, a - radius.

[2] For the cavity filled with a dielectric $\zeta << 1$: $\gamma T << \zeta^2 \pi$.

[3] In [2] it was shown, that at deviation of the initial velocity V_o from the exact synchronism ($V_o = V_{ph}$) $E_{\max}$ increases more than by one order of magnitude, see also [3, 4].

[4] In particular, when the electromagnetic wave is excited by the oscillator beam in the infinite space, with a rather high beam density, and the amplitude of the field being excited is so large that the cyclotron resonances are overlapped, than this amplitude is settled at the level, which is defined by a criterion of overlapping [5].

2 EQUATIONS, CALCULATION

To consider the "short" cavity excitation the given field approximation is sufficient [3, 4]. The equations of beam particle movement in the field of standing cavity wave (n=1) are as follows:

$$\ddot{\xi} = -\mu_1 \cos(\tau - \xi / V_{ph}) - \mu_2 \cos(\tau + \xi / V_{ph}), \quad (1)$$

where $\tau = \omega_1 t$, $\xi = \omega_1 z / V_o$, $\Phi = \tau - \xi / V_{ph}$, $\mu_j = |e| E_j / (m\omega_1 V_o)$, $V_{ph} = \omega_1 / (k_1 V_o)$, $v = V_z / V_o$. Assuming that the Cherenkov's resonance condition is not satisfied the solutions can be presented as $\xi = v_o \tau + \hat{\xi} + \tilde{\xi}$, where $\hat{\xi}$ is slow, $\tilde{\xi}$ is fast. The perturbation theory gives:

$$\ddot{\hat{\xi}} = -\left\langle \sum_{k,p} W_{k,p} \cos\left[G_{k,p}\tau + (k+p)\frac{\hat{\xi}}{V_{ph}} + (k+p-1)\frac{\pi}{2} \right] \right\rangle$$

$$W_{k,p} = \mu_1 J_{k-1}(a) J_p(b) + \mu_2 J_k(a) J_{p-1}(b),$$

$$a = \mu_1 / (V_{ph}\Omega_-^2),\ b = \mu_{21} / (V_{ph}\Omega_+^2),$$

$\Omega_\pm = (v_o \pm V_{ph}) / V_{ph}$, $G_{k,p} = k\Omega_- + p\Omega_+$. Under the resonance conditions $G_{k,p} = 0$, ($k \neq 0$, $p \neq 0$) the equation for $\hat{\xi}$ becomes as a mathematical pendulum equation, for half-width we obtain: $\Xi_{k,p} = 2\sqrt{W_{k,p} V_{ph} / (k+p)}$. If $p = 0$, $v_o = V_{ph}$, then $\Xi_{k,o} = 2\sqrt{\mu_1 V_{ph} J_o(\mu_2 / 4V_{ph})}$. The distance between resonances is $\delta^{k,p}_{k',p'} = V_{ph}\left| \frac{k-p}{k+p} - \frac{k'-p'}{k'+p'} \right|$. The resonances are overlapped if $\Xi_{k,p} + \Xi_{k',p'} > \delta^{k,p}_{k',p'}$. The results solving (1) are presented as a Poincare section in Fig. 1-6 ($\mu_1 = \mu_2$). It is seen that at relatively small field amplitude values the parametric resonances are overlapped and large stochastic regions are arising.

At small field amplitudes when parametric resonances are not overlapped the beam particles cannot occur in the phase space where $v < 0$. They leave the cavity through the output end. In the process of interaction between the continuously injected beam and the cavity field, its amplitude is growing that cause the phase space area increasing where the particle dynamics is not regular. In this case a part of particles which are got this region increases. The beam particles being in the phase space with nonregular dynamics can leave cavity through the input end and get the phase space area where $v < 0$. The stochastic instability is essential if the correlation split time $\tau_c = \left[2\delta^{k,o}_{k',p'} \ln\left(\frac{\Xi_{k,0} + \Xi_{k',p'}}{\delta^{k,o}_{k',p'}} \right) \right]^{-1}$ is more less then the time of flight of beam particles through the cavity T=π V_{ph}. Here the Cherenkov's resonance is chosen as a main one, the distance between the main resonance and neighboring parametric resonances k', p' is chosen as a resonance distance. When τ_c << T the beam particles "forget" about the initial phase, do not participate in the coherently emitting bunches. Besides, the spread of beam particle velocities increases (stochastic heating). This results in decreasing of the efficiency of beam interaction with the cavity field. In this case the large group of particles must leave the cavity across the input end. At τ_c ≈ T one should expect the appearance of back particles. When τ_c ≈ T the field amplitudes correspond to beginning of distortion of the bunches formed due to interaction with the bunch wave, that leads to the fast electron efficiency decreasing.

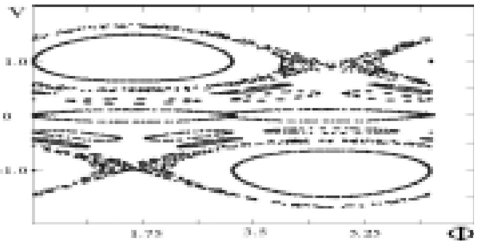

Fig. 1. n=1, V_{ph}=1.01, μ =0.1.

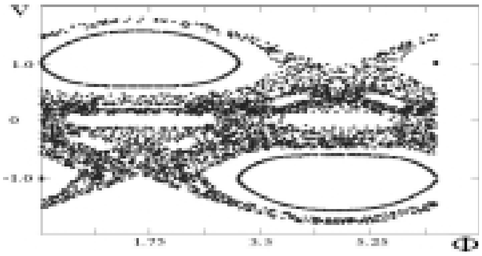

Fig. 2. n=1, V_{ph}=1.01, μ =0.14.

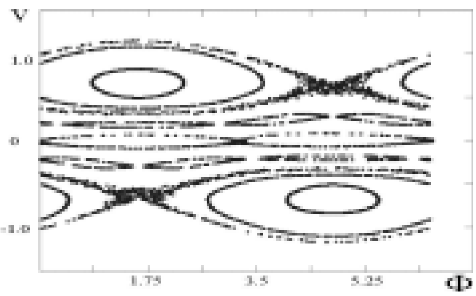

Fig. 3. n=1, V_{ph}=0.7, μ =0.067.

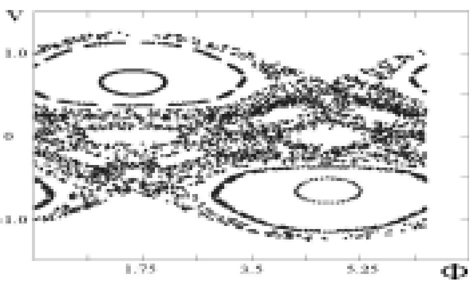

Fig. 4. n=1, V_{ph}=0.7, μ =0.1.

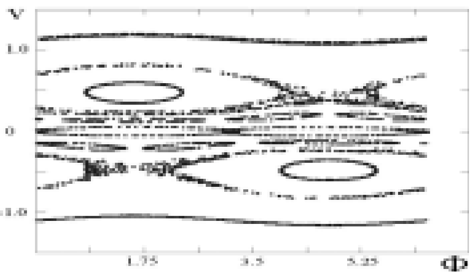

Fig. 5. n=1, V_{ph}=0.5, μ =0.05.

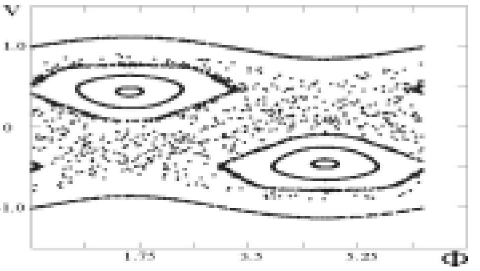

Fig. 6. n=1, V_{ph}=0.5, μ =0.1.

Numerical simulation confirms this picture. Solution (1) was made by the Dorman-Pierce method with step choosing for 200 particles with identical initial velocities. The particles are uniformly placed along the phases for different mode numbers n = {1, 2, 3, 4, 5}. The calculation result is the electron efficiency as a function of μ (Fig. 7, n=1). The vertical lines in this figure point out the field amplitudes when back particles are appeared, digits point out V_{ph}. At V_{ph}=0.8 the maximal electron efficiency value ~ 0.13 is reached at μ =0.198. At μ = 0.24 the back particles appear. At μ >0.27 the electron efficiency becomes negative, i.e., the beam particles in average are accelerated by the given cavity field. The value μ =0.27 corresponds to the maximum field amplitude which can exist in the cavity without losses as a result of interaction with the beam.

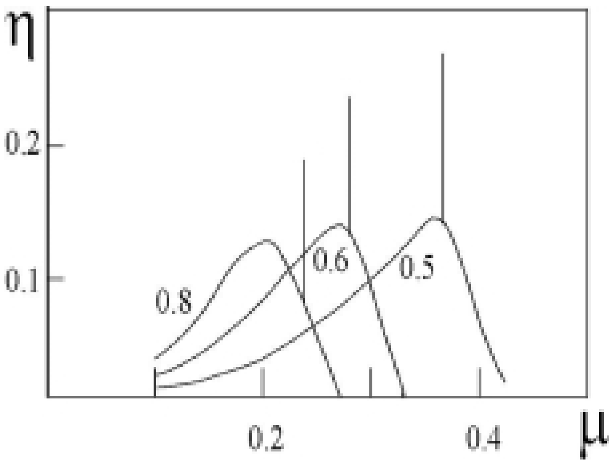

Fig. 7. n=1. Electron efficiency versus field amplitude.

In this case the dynamics of some group of particles is nonregular, a large group of particles leave the cavity through the input end. At V_{ph}=0.60 a maximal electron efficiency of ~ 0.14 is reached at μ =0.277. The back particles appear at μ =0.283. At μ >0.33 the electron efficiency is negative. At V_{ph}=0.50 the maximal value of electron efficiency is reached when μ =0.37. The back particles appear at μ =0.373. The field amplitudes at which the back particles appear can be estimated from the condition $\tau_c \approx T$, where τ_c is defined at k' = p' = 1. In this case the following resonances are overlapped: Cherenkov's resonance for straight wave, parametric resonance for which $\nu = 0$, Cherenkov's resonance on the back wave. These estimations coincide with simulation results. The back particles, as in the case of interaction with a primary mode, in interaction with individual second, third and fourth modes, appear at field amplitudes when Cherenkov's and parametric resonances are overlapped and nonlinear resonances for straight and back waves are overlapped too. However, in this case the Cherenkov's resonances are overlapped weakly, correlation split time due to Cherenkov's resonances overlapping exceeds the time of particle flight through the cavity, while the correlation split time due to overlapping the Cherenkov's and parametric resonances at k'=p'=1 approximately equals the time-of-flight.

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