EFFECTS OF VORTICES ON ION BEAM FOCUSING IN A PLASMA LENS

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For high-current ion-beam focusing a vortical turbulence has been excited in plasma lens by the nonremovable gradient of an external magnetic field. The paper presents theoretical investigation, in the cylindrical approximation, of excitation of slow and fast vortices with taking into account the finite length of plasma lens. It is shown that the growth rate of vortix excitation decreases with decreasing the length of plasma lens. The spatial structures of the vortices are constructed. The expression for the vortex amplitude of saturation is derived.
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1 INTRODUCTION

Excitation of vortices in a plasma lens has been investigated analytically. The plasma lens is designed for ion-beam focusing [1]. A focusing electric field is created by the electron cloud. The cloud density exceeds the ion beam density approximately by 10%. Electrons are distributed on radius approximately homogeneously. The lens represents the cylinder of a finite length, placed in a short coil magnetic field.

As shown in [1], the plasma lens is unstable concerning excitation of oscillating fields. The oscillation excitation is realised owing to a positive radial gradient of a short coil magnetic field.

As in the lens the crossed configuration of radial focusing electric, E_{or}, and longitudinal magnetic fields, H_o, is created, the electrons drift through an angle, θ, with a velocity $V_{\theta o}=-eE_{or}/m_e\omega_{He}$, $\omega_{He}=eH_o/m_ec$.

The density perturbation of primary homogeneous electrons results in appearance of an electric field near the perturbation. Therefore, near the perturbation the crossed fields are realised. Thus, electron dynamics in a field of perturbation is vortical.

In this paper a spatial structure and excitation of vortical perturbations in a plasma lens are investigated theoretically.

2 INSTABILITY DEVELOPMENT IN PLASMA LENS

We use the hydrodynamic and Poisson equations

$$\partial_t\mathbf{V}+(\mathbf{V}\nabla)\mathbf{V}=(e/m_e)\nabla\varphi+[\boldsymbol{\omega}_{\mathbf{He}},\mathbf{V}]-(V^2_{th}/n_e)\nabla n_e \quad (1)$$

$$\partial_t n_e+\nabla(n_e\mathbf{V})=0,\ \nabla\varphi\equiv\nabla\phi-\mathbf{E}_{or},\ \Delta\varphi=4\pi(en_e-q_in_i) \quad (2)$$

From (1) it is possible to derive equations

$$d_t(\alpha-\omega_{He})/n_e=[(\alpha-\omega_{He})/n_e]\partial_zV_z,\ d_tV_z=(e/m)\partial_z\varphi \quad (3)$$

$$d_t=\partial_t+(\mathbf{V}_\perp\nabla_\perp),\quad \alpha\equiv\mathbf{e}_z\mathrm{rot}\mathbf{V}$$

From (1) one can obtain

$$\mathbf{V}_\perp=(e/m\omega_{He})[\mathbf{e}_z,\nabla\varphi]-\omega^{-1}_{He}\partial_t[\mathbf{e}_z,\mathbf{V}_\perp]-\omega^{-1}_{He}[\mathbf{e}_z,(\mathbf{V}\nabla)\mathbf{V}_\perp]\approx$$
$$\approx(e/m\omega_{He})[\mathbf{e}_z,\nabla\varphi]+(e/m\omega^2_{He})\partial_t\nabla_\perp\varphi, \quad (4)$$

$$\alpha\approx 2eE_{ro}/rm\omega_{He}+(eE_{ro}/m)\partial_r(1/\omega_{He})+(e/m\omega_{He})\Delta_\perp\phi+$$
$$+(e/m)(\partial_r\phi)\partial_r(1/\omega_{He})+(e/m)\partial_t\mathbf{e}_z[\nabla,\omega^{-2}_{He}\nabla\phi]. \quad (5)$$

From (2), (5) it approximately follows, $\alpha\approx(\omega^2_{pe}/\omega_{He})\delta n_e/n_{eo,}$ that the vortical motion begins, as soon as there appears a perturbation δn_e.

From (3) one can derive

$$d_t\omega_{He}/n_e=(\omega_{He}/n_e)\partial_zV_z. \quad (6)$$

Taking into account the ion effect, from (2) one can obtain

$$\beta\Delta\varphi/4\pi e=\delta n_e,\ \beta=1-\omega_{pi}^2/(\omega-k_zV_{ib})^2,\ n_e=n_{oe}+\delta n_e. \quad (7)$$

At first let us consider instability development. We search the following dependence $\delta n_e\propto\exp(ik_zz+il_\theta\theta)$. Then from (3) we derive

$$d_t(\omega_{He}/n_e)=-(e\omega_{He}/m_en_{eo})ik_z^2\varphi/(\omega-l_\theta\omega_{\theta o}),\ \omega_{\theta o}=V_{\theta o}/r. \quad (8)$$

From (4), (7), (8) we obtain the equation for ϕ

$$(\omega^2_{pe}/\omega^2_{He})\nabla_\theta\phi\partial_r\omega_{He}+\beta(\partial_t\Delta\phi+\omega_{\theta o}\partial_\theta\Delta\phi)=$$
$$=ik_z^2\phi\omega^2_{pe}/(\omega-l_\theta\omega_{\theta o}). \quad (9)$$

We obtain from (9) dispersion relation, describing the instability development

$$1-\omega^2_{pi}/(\omega-k_zV_{bi})^2-\omega^2_{pe}(l_\theta/r)\partial_r(1/\omega_{He})/k^2(\omega-l_\theta\omega\theta_o)-$$
$$-\omega^2_{pe}k_z^2/k^2(\omega-l_\theta\omega\theta_o)^2=0. \quad (10)$$

Let us take into account that the beam ions pass through the plasma lens during $\tau_i=L/V_{bi}$ and electrons are renovated during $\tau_e>\tau_i$. (10) can be presented as

$$1-\omega^2_{pi}/(\omega-i/\tau_i-k_zV_{bi})^2-\omega^2_{pe}k_z^2/k^2(\omega-i/\tau_e-l_\theta\omega\theta_o)^2-$$
$$-\omega^2_{pe}(l_\theta/r)\partial_r(1/\omega_{He})/k^2(\omega-i/\tau_e-l_\theta\omega\theta_o)=0. \quad (11)$$

Let us mean the quick perturbations for which the phase velocity $V_{ph}\approx V_{\theta o}$. For them from (11) we derive in approximation $k_z=0$, $\omega=\omega^{(o)}+\delta\omega$, $|\delta\omega|<<\omega^{(o)}$ and neglecting τ_e, τ_i

$$\omega^{(o)}=\omega_{pi}=l_\theta\omega\theta_o,\ \omega\theta_o=(\omega^2_{pe}/2\omega_{He})(\Delta n/n_{oe}),\ \delta\omega=i\gamma_q$$
$$\gamma_q=(\omega_{pe}/k)[(\omega_{pi}/2)(l_\theta/r)|\partial_r(1/\omega_{He})|]^{1/2}. \quad (12)$$

From (12) it follows

$$l_\theta=(m_i/m_e)^{1/2}(\omega_{He}/\omega_{pe})(n_{oe}/\Delta n), \quad (13)$$

that for typical parameters of experiments the perturbations with $l_\theta>1$ are excited at a large magnetic field and small electron density.

As γ_q grows with r, taking into account τ_e, τ_i, it can lead to that the perturbations with r smaller than some value can not be excited.

For slow perturbations fulfilled is $V_{ph}<<V_{\theta o}$. We derive for them from (11), in approximation $k_z=0$ and neglecting τ_e, τ_i, the following expressions

$$\gamma_s=(\sqrt{3}/2^{4/3})[\omega^2_{pi}l_\theta(\omega^2_{pe}/2\omega_{He})(\Delta n/n_{oe})]^{1/3}$$
$$k^2=-(r/\omega\theta_o)\omega^2_{pe}\partial_r(1/\omega_{He}),\ \mathrm{Re}\omega_s=\gamma_s/\sqrt{3}. \quad (14)$$

Here γ_s is the growth rate of slow perturbation excitation. As γ_s grows with r, because k grows with r, taking into account τ_e, τ_i it can lead to that the perturbations

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with r smaller than some value are not excited.

From (10) we obtain the growth rate with $k_z \neq 0$

$$\gamma_s=(\sqrt{3}/2^{4/3})\omega^{2/3}_{pi}(l_\theta\omega\theta_o-k_zV_{bi})^{1/3}\times$$
$$\times\{1-k_z^2/[2k_z^2+(l_\theta/r)(l_\theta\omega\theta_o-k_zV_{bi})|\partial_r(1/\omega_{He})|)]\}^{1/3}. \quad (15)$$

From (15) one can see that the particle longitudinal dynamics results in reduction of γ_s. Perturbations with least $k_z \approx \pi/L$ have a maximum γ_s.

3 STRUCTURE OF VORTEX

Let us describe structure of a quick vortex. Neglecting nonstationary terms, we have from (4)

$$\mathbf{V}_\perp=-(e/m_e\omega_{He})[\mathbf{e}_z,\mathbf{E}_{ro}]+(e/m\omega_{He})[\mathbf{e}_z,\nabla\phi], \quad (16)$$
$$V_r=-(e/m_e\omega_{He})\nabla_\theta\phi\ ,\ V_\theta=V_{\theta o}+(e/m_e\omega_{He})\nabla_r\phi,$$
$$V_{\theta o}=-(e/m_e\omega_{He})E_{ro}=(\omega^2_{pe}/2\omega_{He})(\Delta n/n_{oe})r. \quad (17)$$

V_θ can be presented $V_\theta=V_{ph}+\delta V_\theta$. As V_θ equals $V_\theta=rd\theta/dt$, we present $d\theta/dt=d\alpha/dt+\omega_{ph}$, where $\omega_{ph}=(\Delta n/n_{oe})(\omega^2_{pe}/2\omega_{He})|_{r=rv}$, r_v is a vortex localisation. Then, decomposing $\omega_{He}(r)$ on $\delta r=r-r_v$ near r_v, from (17) we derive

$$d\alpha/dt=-(\omega^2_{pe}/2\omega^2_{He})(\Delta n/n_{oe})\delta r(\partial_r\omega_{He})|_{r=rv}+$$
$$+(e/m_e\omega_{He})\partial_r\phi,\ \ dr/dt=-(e/m_e\omega_{He}r)\partial_\theta\phi. \quad (18)$$

Integrating (18), we obtain the equation, describing oscillatory dynamics of electrons in a vortex field

$$(\delta r)^2-(e/m_e)(n_o/\Delta n_o)(\omega_{He}/\omega^2_{pe})8\phi/r_v(\partial_r\omega_{He})|_{r=rv}=\text{const} \quad (19)$$

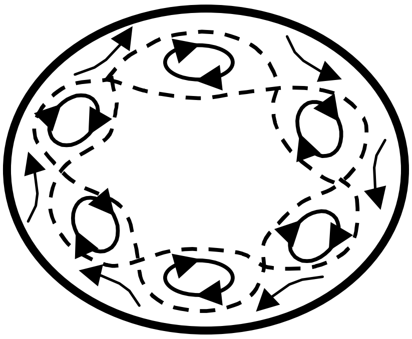

Fig. 1.

Let us consider the following dependence $\phi(\theta)=\phi_o\cos[l_\theta(\theta-\omega_{ph}t)]$ and determine the boundary of a vortex from the condition $\delta r|_{\phi=-\phi o}=0$

$$\delta r=\pm 2[2(e/m)(\phi+\phi_o)(n/\Delta n)(\omega_H/\omega^2_p)/r_v(\partial_r\omega_H)|_{r=rv}]^{1/2}. \quad (20)$$

The radial size of a vortex follows from (20)

$$\delta r_v=2[2(e/m_e)\phi_o(n_{oe}/\Delta n)(\omega_{He}/\omega^2_{pe})/r_v(\partial_r\omega_{He})|_{r=rv}]^{1/2}. \quad (21)$$

From (18), (20) it follows that the frequency of electron oscillations Ω_{tr} on closed trajectories is equal to

$$\Omega_{tr}=(l_\theta/2)(\omega_{pe}/\omega_{He})\times$$
$$\times[2(e/m_e)\phi_o(\Delta n/2n_{oe})(\partial_r\omega_{He})|_{r=rv}/\omega_{He}r_v]^{1/2}. \quad (22)$$

For a simplicity we consider structure of electron trajectories in a slow vortex, $V_{ph} \ll V_{\theta o}$, with $l_\theta=1$ (see. Fig. 2).

From (17) we approximately derive, similarly to (18), equations describing electron oscillations in the vortex field

$$d\theta/dt\approx(\omega^2_{pe}/\omega_{He})(\Delta n/2n_{oe})+(e/m_e\omega_{He}r)\partial_r\phi,$$
$$dr/dt\approx-(e/m_e\omega_{He}r)\partial_\theta\phi. \quad (23)$$

Integrating (23), we obtain

$$r^2+8(e/m_e\omega^2_{pe})(n_{oe}/\Delta n)=\text{const}. \quad (24)$$

Let us consider the following dependence $\phi(\theta)=-\phi_o\cos[l_\theta(\theta-\omega_{ph}t)]$ and determine the boundary of a vortex by putting $r=r_{min}$ at $\phi=\phi_o$. Here r_{min} is a minimum radius of electrons oscillating in a field of a vortex at its boundary. From (24) we derive

$$r=[r^2_{min}+8(\phi_o-\phi)(e/m_e\omega^2_{pe})(n_{oe}/\Delta n)]^{1/2}. \quad (25)$$

From (25) the maximum radius of a vortex is as follows

$$r_{max}=[r^2_{min}+16\phi_o(e/m_e\omega^2_{pe})(n_{oe}/\Delta n)]^{1/2}. \quad (26)$$

From (26) we obtain the radial size, Δr_s, of a vortex

$$\Delta r_s=[r^2_{min}+16\phi_o(e/m_e\omega^2_{pe})(n_{oe}/\Delta n)]^{1/2}-r_{min}. \quad (27)$$

In case $r_{min}=0$

$$\Delta r_s|_{rmin=0}=[16\phi_o(e/m_e\omega^2_{pe})(n_{oe}/\Delta n)]^{1/2}. \quad (28)$$

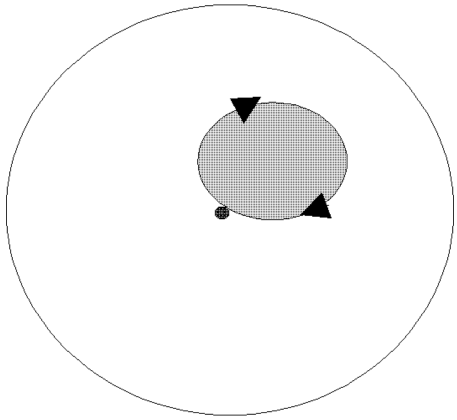

Fig. 2.

Instability is developed in a homogeneous plasma so long as bunching of homogeneously distributed electrons happens. Bunching ceases, when a slow (adiabatic) stage of electron dynamics comes,

$$\Omega_{tr} \geq \gamma. \quad (29)$$

From (12), (22), (29) it follows that it happens at the amplitude of an electrical potential of a fast vortex

$$\phi_{slh}\approx(\omega_{He}\omega_{pi}/2k^2)(n_{oe}/\Delta n). \quad (30)$$

However the amplitudes of a set of separate vortices with a large distance between them can grow further.

The maximum amplitude of this set of vortices, ϕ_{sm}, is determined by a condition, that the magnetic force does not keep any more electrons of a vortex, rotated around its axis on the closed trajectories. In other words the electron bunch of a vortex can extend across a magnetic field. Thus bunching of electrons ceases. Thus from a violation of forces balance,

$$m_eV^2_\theta/r-eE_r \geq m_e\omega_{He}V_\theta. \quad (31)$$

one can obtain the amplitude of a vortex saturation, ϕ_{sm}. Here E_r is the electrical field of a vortex, the perturbation of electron density in which is δn_{ev}. From (31) it follows that electrons of a vortex at inequality fulfilment

$$\omega^2_{pe}(\delta n_{ev}+\Delta n)/n_{oe} \geq \omega^2_{He}/2. \quad (32)$$

can freely move across a magnetic field. Thus, using (2), we have found, that the amplitude of a vortex is stabilised at [1]

$$\phi_{sm}\approx(m_e/ek^2)[\omega^2_{He}/2-(\Delta n/n_{oe})\omega^2_{pe}]. \quad (33)$$

From (33) one can see that if Δn is close to

$$\Delta n\approx H_o^2/8\pi m_ec^2, \quad (34)$$

the vortical perturbations are not excited.

Let us compare ϕ_{slh} with ϕ_{sm}

$$\phi_{slh}/\phi_{sm}\approx 2(n_{oe}/\Delta n)(\omega_{pi}/\omega_{He}). \quad (35)$$

One can see that in case of heavy ions and large magnetic field the inequality $\phi_{slh} \ll \phi_{sm}$ is fulfilled.

4 EXCITATION OF NON-LINEAR VORTICES

Let us describe excitation of non-linear vortices using equations

$$\beta\Delta\varphi=4\pi e\delta n_e,\ \beta=1-\omega^2_{pi}/\omega^2,\ \delta n_e=n_e-n_{eo}, \quad (36)$$

$$d_t(\alpha-\omega_{He})/n_e=0,\ d_t=\partial_t+(\mathbf{V\nabla}), \quad (37)$$

$$\mathbf{V}=-(e/m_e\omega_{He})[\mathbf{e}_z,\mathbf{E}_{ro}]+(e/m\omega_{He})[\mathbf{e}_z,\mathbf{\nabla}\phi]- -\omega^{-1}_{He}\partial_t[\mathbf{e}_z,\mathbf{V}]-\omega^{-1}_{He}[\mathbf{e}_z,(\mathbf{V\nabla})\mathbf{V}]. \quad (38)$$

Selecting a time derivative ∂_τ and a vortex motion in the crossed fields with a velocity $V_{s\theta}$, we have

$$d_t=\partial_\tau+(\mathbf{V\nabla})-V_{s\theta}\nabla_\theta. \quad (39)$$

From (36)-(38) we obtain

$$\alpha\approx-2eE_{ro}/rm_e\omega_{He}-(eE_{ro}/m)\partial_r(1/\omega_{He})+(e/m\omega_{He})\Delta\phi+ +(e/m)(\partial_r\phi)\partial_r(1/\omega_{He})+ +(e/m_e)\partial_t[r^{-1}\partial_r(1/\omega^2_{He})\partial_\theta\phi-r^{-1}\omega^{-2}_{He}\partial^2_{\theta r}\phi], \quad (40)$$

$$d_t(\omega_{He}/n_e)=0,\ n_e=n_{eo}+(q_i/e)\delta n_i+\Delta\phi/4\pi e, \quad (41)$$

$$V_\theta\approx V_{\theta o}+(e/m_e\omega_{He})[\mathbf{e}_z,\mathbf{\nabla}\phi], \quad (42)$$

$$V_{\theta o}=-(e/m_e\omega_{He})E_{ro}=(\omega^2_{pe}/2\omega_{He})(\Delta n/n_{oe})r. \quad (43)$$

From (36), (41), (42) we derive the non-linear evolution equation, describing an excitation of non-linear vortices

$$[\partial_\tau+(\mathbf{V}_{\theta o}-\mathbf{V}_{s\theta})\mathbf{\nabla}_\theta+ +(e/m_e\omega_{He})([\mathbf{e}_z,\mathbf{\nabla}\phi]\mathbf{\nabla})]\omega_{He}/(n_{eo}+(q_i/e)\delta n_i+\Delta\phi/4\pi e)\approx 0. \quad (44)$$

In stationary approximation the slow vortex is described according to (44) by equation

$$[(V_{\theta o}-V_{s\theta})\nabla_\theta+ +(e/m_e\omega_{He})([\mathbf{e}_z,\mathbf{\nabla}\phi]\mathbf{\nabla})](\omega^2_{pe}+\Delta\phi e/m_e)/\omega_{He}\approx 0. \quad (45)$$

The equation (45) can be presented in the form

$$(1-V_{s\theta}/V_{\theta o})\nabla_\theta\Delta\phi-2(n_{oe}/\Delta n)(\nabla_\theta\phi)r^{-1}\partial_r(1/\omega_{He})+ +2(e/m_e\omega_{He}\omega^2_{pe})r^{-1}(n_{oe}/\Delta n)\{\phi,\Delta\phi\}_{r,\theta}=0, \quad (46)$$

$$\{\phi,\Delta\phi\}_{r,\theta}\equiv(\nabla_r\phi)\nabla_\theta\Delta\phi-(\nabla_\theta\phi)\nabla_r\Delta\phi= =r^{-1}[(\partial_r\phi)(\partial_\theta\Delta\phi)-(\partial_\theta\phi)(\partial_r\Delta\phi)].$$

Taking into account in (44) terms with ∂_t and effect of ions we derive

$$\partial_t\Delta\phi\approx-4\pi q_i(V_{\theta o}\nabla_\theta))\delta n_i. \quad (47)$$

One can obtain from the hydrodynamic equations the equation for the ion density perturbation, δn_i,

$$\partial^2_t\delta n_i\approx n_{oi}(q_i/m_i)\Delta\phi. \quad (48)$$

From (47), (48) we obtain the equation, describing the vortex excitation

$$\partial_t^3\Delta\phi\approx-\omega^2_{pi}(V_{\theta o}\nabla_\theta)\Delta\phi. \quad (49)$$

The solution (49) we search as

$$\phi=\phi_o\eta[\theta-\int dt\ \delta\omega_{\theta s}]. \quad (50)$$

Here η is the quasi-stationary shape of the vortex, determined by (46); $\delta\omega_{\theta s}$ is the shift of the angle frequency of the vortex, determined by its interaction with ions.

From (46), (49), (50) one can show that the non-linear growth rate of the vortex excitation is proportional to $\gamma_{NLs}\propto\gamma_s\propto(m_e/m_i)^{1/3}$.

From (44) and hydrodynamic equations for ions in stationary approximation we derive for a quick vortex

$$[(V_{\theta o}-V_{s\theta})\nabla_\theta+(e/m_e\omega_{He})\{\phi,\ \}_{r,\theta}](\Delta\phi+q_i\delta n_i 4\pi)\approx 0. \quad (51)$$

$$\{\phi,\ \}_{r,\theta}=r^{-1}[(\partial_r\phi)\partial_\theta-(\partial_\theta\phi)\partial_r],\ (V_{s\theta}\nabla_\theta)^2\delta n_i\approx n_{oi}(q_i/m_i)\Delta\phi.$$

Taking into account in (44) the term with ∂_τ and radial non-uniformity of ω_{He} we derive the following equation for a quick vortex

$$\partial_\tau\delta n_e\approx(e/m_e)(\nabla_\theta\phi)n_{oe}\partial_r(1/\omega_{He}). \quad (52)$$

Using (41), we rewrite (54) in the following form

$$\partial_\tau\Delta\phi+4\pi q_i\partial_\tau\delta n_i=(\nabla_\theta\phi)\omega^2_{pe}\partial_r(1/\omega_{He}). \quad (53)$$

From the hydrodynamic equations for ions one can obtain the equation

$$(V_{s\theta}\nabla_\theta-\partial_\tau)^2\delta n_i\approx n_{oi}(q_i/m_i)\Delta\phi. \quad (54)$$

From (53), (54) we have, with taking into account that for main terms of (51) the following equality $(V_{s\theta}\nabla_\theta)^2\phi\approx-\omega^2_{pi}\phi$ is approximately true, the equation

$$\partial^2_\tau\Delta\phi=(\omega^2_{pi}/2V_{s\theta})\phi\omega^2_{pe}\partial_r(1/\omega_{He}). \quad (55)$$

Let us introduce a non-linear wave number, κ_{NL}, following the expression $\Delta\phi\equiv-\kappa^2_{NL}\phi$. Then from (55) we obtain the non-linear growth rate of the quick vortex excitation

$$\gamma^{(q)}_{NL}\approx(\omega_{pi}/\kappa_{NL})[(\omega^2_{pe}/2V_{\theta o})\partial_r(1/\omega_{He})]^{1/2}\propto\gamma_q. \quad (56)$$

5 CONCLUSION

So, it is shown that the electron density perturbation in the plasma lens leads to the vortices. Two kinds of vortices are excited: quick perturbations with V_{ph} close to the drift velocity, $V_{ph}\approx V_{\theta o}$, and slow perturbations with $V_{ph}<<V_{\theta o}$. Owing to the fast pass of ion beam through the plasma lens and finite time of electron renovating the perturbations are not excited in the neighbourhood of its axis. The radial width of vortices is proportional to the radical square from the amplitude of the vortex electric potential, $\surd\phi_o$. The radial width of a quick vortex depends also on the radial gradient of a magnetic field. The instability of excitation of a homogeneous vortical turbulence is saturated at a low level, when a frequency of electron oscillations in the vortex on the closed trajectories begins to exceed the growth rate of the instability development. At large amplitudes the set of separated non-linear vortices is excited. The non-linear growth rates are proportional to the linear growth rates.

6 ACKNOWLEDGEMENTS

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