THE NUMERICAL FIELD CALCULATION IN STRUCTURES WITH THE SPACE-UNIFORM QUADRUPLE FOCUSING BY THE METHOD OF AUXILIARY CHARGES

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In the paper the procedure of solving integral equations with the use of auxiliary charges is used. The electrostatic field potential is presented in the form of superposition of the fields of point sources located out of the investigated region at some distance from it. The values of N charges are determined from the boundary conditions at N points of the boundary. The definition of the point sources is equivalent to using some function of the charge density at the boundary of the region.
This approach was used for the calculation of structures with the space-uniform quadruple focusing with sinusoidal and trapezoidal modulation, and also a section of the transverse matching of the "funnel".
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Presently as the initial section of the ion accelerators the structures with the space-uniform quadruple focusing are widely used [1].

The main parameters necessary for calculation of the accelerating and focusing canal of such type are: efficiency of acceleration, T, efficiency of focusing, κ, maximum electric field strength at the electrode surface, E_s [2,3].

In this paper for calculation of these values a version of integral equation technique is used - the method of auxiliary charges (MAC) [4].

According to this method the electrostatic field potential can be represented as a superposition of the point charge fields located outside the examined area and at a certain distance from it. Values of N charges are found from the boundary conditions at N points. The definition of the point charges is equivalent to application of the certain charge density at the electrode surface.

The computational procedure is used in the RFQFLD code. The code gives a possibility to calculate fields in the RFQ structures with semicircular electrodes modulated in the sinusoidal and trapezoidal law and also the fields in the section of the transverse matching ("funnel").

The boundary conditions for the electrostatic field potential $\varphi_o(x,y,z)$ was chosen with taking into account the symmetry

$$\varphi_o(x,y,z) = \varphi_o(x,-y,z) = \varphi_o(-x,y,z)$$

In the accelerating structure of the RFQ type there are 3 types of the boundary conditions:

1. For the section of the transverse matching the boundary conditions are:
   $\varphi_i(z = 0) = 0$,
   $\varphi_i(z = L_p) = 0$,
   where $z = 0$ is the longitudinal coordinate of the cavity wall, L_p is the length of the section of the transverse matching.
2. For the bunching section when it is necessary to take into account the accelerating structure irregularity. When the boundary conditions are setting it is necessary to take into account the influence of the adjoining cells then

$$\varphi_i(z = 0) = 0,\ \varphi_i(z = L_p) = 0.$$

Here $z = 0$ is the cross-section at the point of the maximum electrode deviation from the axis. $L_s = L_{i-1} + L_i + L_{i+1}$; where $L_{i\ i}$ is the length of the accelerating cell in which the field potential is calculated, L_{i-1} and L_{i+1} are the lengths of the neighbor cells.

For the accelerating section where the structure irregularity can be neglected

$$\varphi_i(z = 0) = 0,\ \varphi_i(z = L_i) = 0.$$

Since the cavity walls are far from the area of interest their influence is not taken into account.

Using the specified boundary conditions the field calculation in the RFQ structure could be restricted to calculation in one quadrant with taking into account the influence of the symmetry of the auxiliary charges.

The static potential calculations are carried out for the cell with sinusoidal modulation having the following parameters:

$$kR_o = 1;\ R_e/R_o = 0.7;\ m = 1.8$$

where $k = \pi/L$, L - the accelerating cell length;
R_e - the radius of the electrode rounding;
R_o - the average aperture radius (the distance between the electrode and the axis at the point of the exact symmetry of electrodes;
m - the electrode modulation depth.

In Fig. 1 the field potential generated by the calculated auxiliary charges is shown in the cross-section at the point of accurate RFQ electrode symmetry.

The equipotentials -1 and +1 correspond to the electrode surfaces. The white circles show the arrangement of the auxiliary charges, black ones show the points at the surface where the boundary conditions were defined.

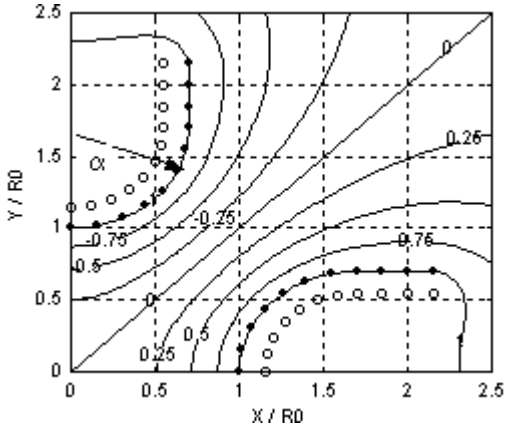

Fig. 1. The field potential generated with the auxiliary charges in the cross-section with the accurate RFQ electrode symmetry.

One of the important features of the MAC is the optimization of the auxiliary charge arrangement with respect to the boundary. It is necessary that the boundary conditions are held as accurately as possible at the all surface points.

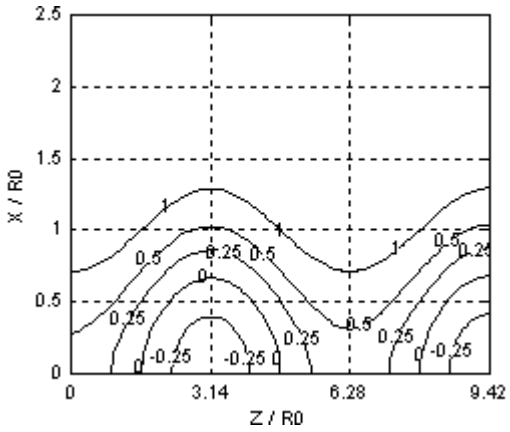

Fig. 2. The field potential generated with the auxiliary charges in the transverse cross-section of the structure.

The preliminary calculations have shown that the distance from the auxiliary charges to the surface must be equal or somewhat larger than the distance between points at the electrode surface where the boundary conditions are set.

With this arrangement the charge discreteness is insignificant at the boundary surface, and the matrix for these charges determination remains quite specified.

In Fig. 2 the field potential generated by the auxiliary charges at the transverse cross-section is shown. The +1 equipotential corresponds to the electrode contour.

In Fig. 3 the longitudinal field on the axis of the structure is presented. The field values in the middle cell are used (the range [3.14, 6.28]).

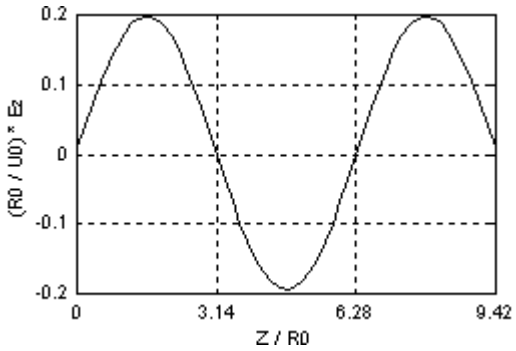

Fig. 3. The longitudinal component distribution at the axis of the structure with the constant modulation depth of electrodes.

In Fig. 4 the factor of the field increment versus the angle coordinate α in the cross-section with the accurate RFQ electrode symmetry is shown.

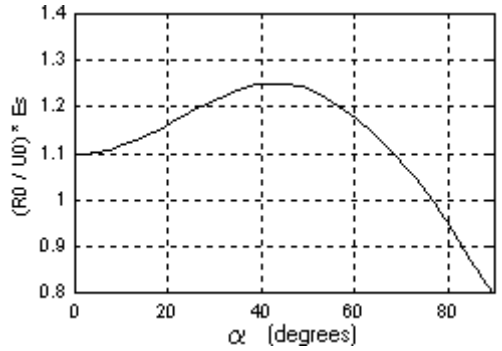

Fig. 4. Distribution of the amplification factor in the cross-section with RFQ symmetry.

Here U_o is the potential difference between the electrodes.

The following ought to be said about this value. At the initial part of the structure ($kR_o \le 1$), χ has the maximum value in the cross-section with the maximum deviation of the electrodes from the axis and depends strongly on the modulation depth m. That leads to the situation that extremely large m at the initial part of the accelerating structure may cause significant loss of the electric strength. With $kR_o > 1$ χ has the maximum value at the point of the accurate RFQ symmetry and depends mainly on the radial fields.

The calculation errors were evaluated in the following way [4]. If φ is a harmonic function approximating the solution φ with the accuracy ε at the boundary Γ, i.e., the condition $|\varphi - \varphi| < \varepsilon$ is fulfilled, then the error of the approximate solution will not exceed ε in any point in the range under consideration. In Fig. 5 the relative error $\delta = \dfrac{\tilde{\varphi} - \varphi}{\varphi}$ versus the angle coordinate α is shown. The auxiliary charge arrangement was chosen so that the presented dependence reflects the maximum error in the boundary conditions. As one can see from Fig. 5, the maximum error of $1.75 \cdot 10^{-1}$ occurs for

$\alpha \cong 85^{o}$, however, this part of the electrode practically does not influence the field distribution in the region of the calculations. Therefore we can believe that the error for the field potential does not exceed $1 \cdot 10^{-3}$.

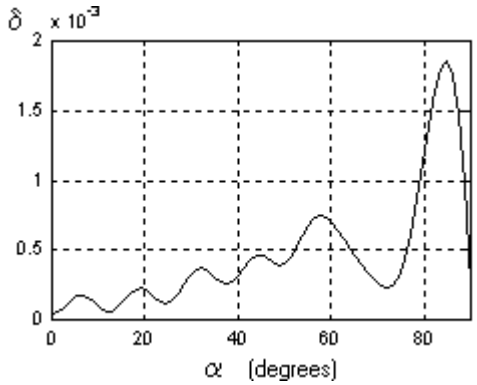

Fig. 5. The relative error of the boundary condition approximation in the cross-section with RFQ symmetry.

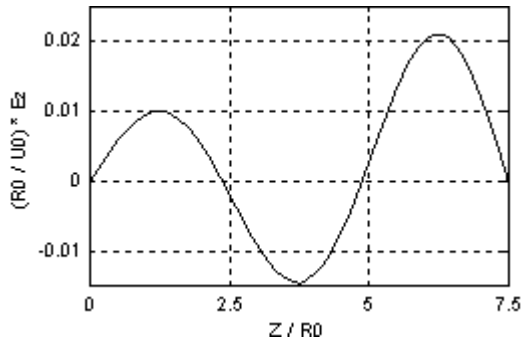

Fig.6. Distribution of the longitudinal component for the structure with alternating depth of the electrode modulation.

In Fig. 6 the longitudinal field distribution on the axis of the structure calculated with the method of the auxiliary charges for the bunching section with taking into account the irregularity of the accelerating structure is shown.

In conclusion we will note that the MAC method is extremely simple from the viewpoint of programming and calculations.

It took 15.2 s to calculate T, κ, χ_s, δ with the Pentium 150 MHz. The number of the auxiliary charges in the region of calculations (without symmetric reflections) was 2 x 11 x 20 = 440.

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