INFLUENCE OF POLARIZATION OF A REGULAR ELECTROMAGNETIC WAVE ON STOCHASTIC ACCELERATION OF CHARGED PARTICLES IN AN EXTERNAL MAGNETIC FIELD

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1 INTRODUCTION

One of the ways of realization of the "channel" of direct field energy transmission to particles is creation of conditions, at which the motion of particles in a field of regular waves becomes chaotic. In the present report the chaotic dynamics of charged particles in the field of regular electromagnetic wave propagating at the angle to the constant external magnetic field is considered. The stochastic instability develops as a result of overlapping of nonlinear cyclotron resonances. We have found conditions at which the particle, which is got into regime of stochastic acceleration, remains in it without reference to energy, which the particle is gained during the interaction with wave. It is shown that these conditions essentially depend on wave polarization. So if in wave with circular or H-polarization the "unlimited" stochastic acceleration is possible, in wave with E-polarization the "unlimited" stochastic acceleration is impossible.

2 STATEMENT OF A PROBLEM. THE BASIC EQUATIONS

We consider the motion of a charged particle in constant externally applied magnetic field $\vec{H} = \{0,0,H_o\}$ and in the field of an electromagnetic plane wave, propagating at the angle ϕ to the field $\vec{H}$:

$$\vec{E} = \mathrm{Re}\{E_o\vec{\alpha}\exp(i\vec{k}\vec{r} - i\omega t)\},$$
$$\vec{H} = \mathrm{Re}\{c/\omega[\vec{k}\vec{\alpha}]E_o\exp(i\vec{k}\vec{r} - i\omega t)\} \quad (1)$$

Here E_o - wave amplitude, $\vec{\alpha} = \{\alpha_x, i\alpha_y, \alpha_z\}$ - polarization vector of the wave, wave vector $\vec{k} = \{\omega/c\cdot N\sin\phi, 0, \omega/c\cdot N\cos\phi\}$ has only two varieble*s* choice of coordinates. N - is the index of refraction. In dimensionless variables ($t\to\omega t$, $\vec{r}\to\vec{r}\omega/c$, $\vec{p}\to\vec{p}/mc$, $\vec{k}\to\vec{k}c/\omega$) the equations of particle motion can be reduced to the form:

$$\dot{\vec{p}} = (1-\vec{k}\vec{p}/\gamma)\,\mathrm{Re}(\vec{E}e^{i\psi}) + (\omega_h/\gamma)[\vec{p}\vec{h}] + \vec{k}/\gamma\,\mathrm{Re}\{(\vec{p}\vec{E})e^{i\psi}\}, \quad (2)$$
$$\dot{\vec{r}} = \vec{p}/\gamma\ ,\quad \dot{\psi} = \vec{k}\vec{p}/\gamma - 1,$$

where $\psi = \vec{k}\vec{r} - t$, $\vec{h} = \vec{H}/H_o, \omega_h = eH_o/mc\omega$, $\vec{E} = eE_o\vec{\alpha}/mc\omega$, $\gamma = (1+\vec{p}^2)^{1/2}$ - particle energy, $\vec{p}$ - its momentum. The set of Eqs. (2) possesses the integral of motion:

$$\vec{p} - \mathrm{Re}(i\vec{E}e^{i\psi}) + \omega_h[\vec{r},\vec{h}] - \vec{k}\gamma = const\,. \quad (3)$$

For subsequent analysis, it is convenient in (2), (3) to transform to new variables $p_\perp, p_z, \theta, \xi, \eta$ - guiding center coordinates, by formulas:

$$p_x = p_\perp\cos\theta,\quad p_y = p_\perp\sin\theta,\quad p_z = p_z,$$
$$x = \xi - p_\perp/\omega_h\sin\theta, y = \eta + p_\perp/\omega_h\cos\theta\,, \quad (4)$$

Suppose that the amplitude of the electromagnetic field $\varepsilon_o = eE_o/mc\omega$ is sufficiently small and taking into account that the particle will interact efficiently with the wave if it fulfills one of the resonance conditions:

$$k_z p_z + s\omega_h - \gamma = 0,\quad s = \ldots,-2,-1,0,1,2,\ldots \quad (5)$$

after averaging over the fast time scale one can find following equations, which describe particle motion in the case of an isolated resonance:

$$\dot{p}_\perp = (1-k_z v_z)W_s\varepsilon_o\cos(\theta_s)/p_\perp$$
$$\dot{p}_z = k_z W_s\varepsilon_o\cos(\theta_s)/\gamma,\ \dot{\gamma} = W_s\varepsilon_o\cos(\theta_s)/\gamma, \quad (6)$$
$$\dot{\theta}_s = k_z v_z - 1 + s\omega_h/\gamma + \varepsilon_o F_s\sin(\theta_s)\,,$$

where

$$F_s = s(1-k_z v_z)/p_\perp(\alpha_y sJ_s(\mu)/\mu - \alpha_x J_s'(\mu)) + \frac{k_x^2 v_\perp}{\omega_h}\frac{\alpha_y s}{\mu}J_s'(\mu) - \frac{sk_x}{p_\perp}(\frac{\alpha_y p_\perp}{\gamma}J_s(\mu) - \alpha_z v_z J_s'(\mu)) + (k_x\alpha_y/\omega_h)(1-k_z v_z)J_s(\mu)\,,$$
$$W_s = \alpha_x p_\perp sJ_s(\mu)/\mu - \alpha_y p_\perp J_s'(\mu) + \alpha_z p_z J_s(\mu)\,,$$

$\mu = k_x p_\perp/\omega_h$, $J_s(\mu)$ - the Bessel function, $J_s'(\mu)$ - its derivative over the argument.

3 CONDITIONS OF NONLINEAR RESONANCES OVERLAPPING

Lets suppose, that during the particle interaction with the wave its energy varies a little, i.e. $\gamma = \gamma_{0s} + \tilde{\gamma}_s, \tilde{\gamma}_s << \gamma_{0s}$, where γ_{0s} meets the resonant condition (5) and in view of the approximate integral of motion:

$$p_z - k_z\gamma = a = const\ , \quad (7)$$

a closed set of equations for $\dot{\theta}_s$ and $\tilde{\gamma}_s$ can be obtained from (3):

$$\dot{\tilde{\gamma}}_s = W_s\varepsilon_o\cos(\theta_s)/\gamma_{0s}\,, \dot{\theta}_s = (k_z^2 - 1)\tilde{\gamma}_s/\gamma_{0s}\,. \quad (8)$$

Equations (8) are the equations of the mathematical pendulum. It is easy to find the width of the nonlinear isolated resonance from these equations:

$$\Delta\tilde{\gamma}_s = 2\sqrt{\left|\varepsilon_o W_s/(1-k_z^2)\right|}\ . \quad (9)$$

One ought to notice, that when we derived the equations (8) we neglected the addend $\varepsilon_o F_s\sin(\theta_s)\propto\varepsilon_o$, assuming it small. It is never correct to do this, when F_s includes the components, which are proportional to $1/p_\perp$ and, under condition that $p_\perp$ tends to zero, it is necessary to take it into account. The distance between adjacent resonances was found with the help of the resonant conditions (5) and the approximate integral of motion (7):

$$\delta\tilde{\gamma}_s = \omega_h / (1-k_z^2) \qquad (10)$$

Relations (9), (10) allow one to put down the generalized Chirikov's criterion:

$$\varepsilon_o > \omega_h^2 / 16W_s(1-k_z^2) \qquad (11)$$

of the development of the local instability of charged particles motion under their interaction with the electromagnetic wave in an external magnetic field.

The width of the nonlinear resonance $\Delta\tilde{\gamma}_s$ under fulfilment the condition (11) becomes greater than the distance between the neighboring resonances $\delta\tilde{\gamma}_s$. The overlapping of the nonlinear resonances takes place. Then the dynamics of particles becomes chaotic.

Let us suppose, that the particle is in resonance with the number $s = s^*$ and the amplitude of the field is those, that the stochastic instability of charged particle motion takes place. In space ($\gamma, p_\perp, p_z$) particle motion is determined by approximate integral (7) and resonace conditions (5), which in the plane ($p_\perp, p_z$) become:

$$\frac{p_\perp^2}{\dfrac{s^2\omega_h^2}{(1-k_z^2)} - 1} + \frac{\left(p_z - \dfrac{k_z s\omega_h}{(1-k_z^2)}\right)^2}{s^2\omega_h^2 - (1-k_z^2)} = 1, \; k_z^2 \neq 1,$$

$$p_\perp^2 - 2s\omega_h p_z = s^2\omega_h^2 - 1, \qquad k_z^2 = 1, \qquad (12)$$

$$p_\perp^2 + s^2\omega_h^2 (p_z - k_z s\omega_h / (1-k_z^2))^2 = 0,$$

$$s^2\omega_h^2 = 1 - k_z^2.$$

Particle, remaining on the integral (7) diffuses on resonances (12). To estimate the resonance width with number which greatly exceeding s^*, we find constant a, which is the part of (7). By substituting in integral (7) value of energy (5) we take:

$$a(s^*) = (1-k_z^2) p_{zs^*} - k_z s^* \omega_h. \qquad (13)$$

Value p_{zs^*} we determine from the first equation (12):

$$p_{zs^*} = \frac{s^*\omega_h k_z}{1-k_z^2} \pm \sqrt{\frac{1}{1-k_z^2}\left\{\frac{s^{*2}\omega_h^2}{1-k_z^2} - 1 - p_{\perp s^*}^2\right\}} \qquad (14)$$

Substituting p_{zs^*} from (13) into (12) we find:

$$a(s^*) = \pm\sqrt{\left\{s^{*2}\omega_h^2 - (1-k_z^2)^2(1-p_{\perp s^*}^2)\right\}} \qquad (15)$$

Let us suppose now, that the particle has got in the vicinity with number $n >> s^*$. To find width of this resonance it is necessary to calculate p_{zn} and $p_{\perp n}$.

Using resonance condition and first equation from (12) we find:

$$p_{zn} = (a(s^*) + k_z n\omega_h) / (1-k_z^2), \; p_{\perp n} = \frac{n\omega_h}{k_z}. \qquad (16)$$

4 INFLUENCE OF ELECTROMAGNETIC WAVE POLARIZATION ON CHARGED PARTICLES' ACCELERATION REGIME

Estimate width of the nonlinear (9) for wave with circular polarization ($\vec{k} = \{\sin\phi, 0, \cos\phi\}$, $\vec{\alpha} = \{-\cos\phi, i, \sin\phi\}$) when $n >> s^*$:

$$\Delta\tilde{\gamma}_n = 4\sqrt{\varepsilon_o \omega_h c_1 n^{1/3} / \sin^3\phi}, \; (c_1 = 0.411) \qquad (17)$$

and separation between adjacent resonances (10):

$$\delta_{\gamma_{n,n+1}} = |\omega_h| / \sin^2\phi. \qquad (18)$$

Using (17) and (18) we can write the criterion of development of the stochastic instability of charged particles motion as:

$$16\varepsilon_o c_1 n^{1/3} \sin\phi / \omega_h > 1. \qquad (19)$$

From (19) it is seen, that the more n, the better fulfilled inequation (18). It is connect to that width of nonlinear resonace (18) increase with growth of number n. Condition (19) is criterion of the particle caption into regime of "unlimited" stochastic acceleration. At given field amplitude, the number of resonance, the particle is in, and cyclotron frequency, (19) allows to estimate the angle range ϕ, at which "unlimited" stochastic acceleration is possible. From (19) it is seen, that transversal propagation of wave with respect to magnetic field is optimal for stochastic acceleration.

For wave with linear polarization of type: $\vec{\alpha} = \{0, i, 0\}$ (M-wave) condition (19) does not vary. If polarization has type $\vec{\alpha} = \{-\cos\phi, 0, \sin\phi\}$ (E-wave), the width of nonlinear resonance decreases with growth of number n:

$$\Delta\tilde{\gamma}_n = 4\sqrt{\varepsilon_0 a(s^*) c_0 n^{-1/3} / \sin^3\phi}, \; (c_0 = 0.477), \qquad (20)$$

and criterion (11) is possible to represent as:

$$16\varepsilon_o c_o a(s^*) n^{-1/3} \sin\phi / \omega_h^2 > 1. \qquad (21)$$

From (21) it is seen that at big n criterion (21) ceases to be fulfilled. It means, that the diffusion of particles in high-energy region through development of stochastic instability of charged particles motion due to overlappings of nonlinear cyclotron resonances (5) is impossible. The strong dependence from polarization of wave is explained by structure of function W_s, with which the width of nonlinear resonance is determined. If $\alpha_y \neq 0$, at s >> 1 the width of nonlinear resonance is determined by the second term in W_s, which at s >> 1 gives: $W_s \propto s^{1/3}$. In the case E - wave, when $\alpha_y = 0$, the width of nonlinear resonance is determined by the first and last terms in W_s, which combination at s >> 1 gives: $W_s \propto s^{-1/3}$.

5 NUMERICAL ANALYSIS RESULTS

In order to demonstrate possibility of "unlimited" stochastic acceleration of charged particles by electromagnetic wave with polarization $\vec{\alpha} = \{-\cos\phi, i, \sin\phi\}$, propagating on the angle ϕ to $\vec{H}_o$, it was numerically solved equations (2) for assembly out of 400 particles, uniformly distributed on phase ψ , c by identical values γ_o=1.794 and p_{zo}=0.5. Refractive index N=1. Angle $\phi = 0.3\pi$ is chosen so, that all particles in initial point of time are in resonance $s^* = 3$. Amplitude of field $E_o = 0.2$, cyclotron frequency $\omega_h = 0.5$. The similar calculations were carried out and for the wave with polarization $\vec{\alpha} = \{-\cos\phi, 0, \sin\phi\}$.

The precision of calculations was checked with the help of integral (5) and did not exceed 10^{-4}. At the chosen parameters the particles in initial point of time are moving nonregularly. Dependence amplitude of the field E_* at which the resonances with the numbers s and s+1 are overlapped, from the numbers of resonance s, for the wave with circular polarization $\vec{\alpha} = \{-\cos\phi, i, \sin\phi\}$ is given on Fig. 1 (a), and for E-wave with polarization $\vec{\alpha} = \{-\cos\phi, 0, \sin\phi\}$ on Fig. 1(b).

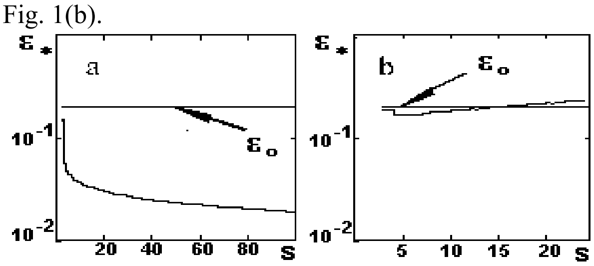

Fig. 1. Dependence of amplitude of overlapping resonances E_* from resonance number s.

It is clear, that for the circular polarized wave with the growth of particle energy the condition of resonance overlapping is fulfill better and better.

Maximum value of the number of resonance for E-wave is s=16 ($\gamma_{16} \approx 11$), above this criterion of overlapping nonlinear cyclotron resonances of first order is inadequate. On Fig. 2 there is projection of particle trajectory with initial phase $\psi = 0$ to plane (γ, p_z) under interaction with circular polarized wave $\vec{\alpha} = \{-\cos\phi, i, \sin\phi\}$, which is approximate integral (7).

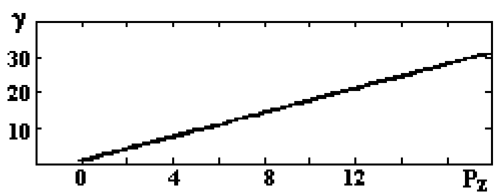

Fig.2. Projection of particle trajectory to plane (γ, p_z).

One can see, that approximate integral is conserved. On Fig. 3-4 the time dependences of particle energy and maximum Lyapunov index for wave with polarization $\vec{\alpha} = \{-\cos\phi, i, \sin\phi\}$ are represented. It is seen that particle dynamics is nonregular.

The similar dependences, but for particle moving in the field of E-wave are shown on Fig. 5-6.

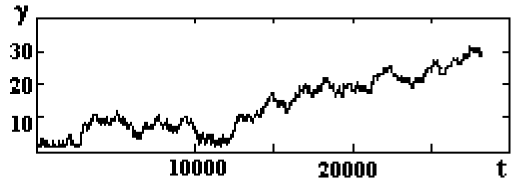

Fig. 3. Time dependence of particle energy.

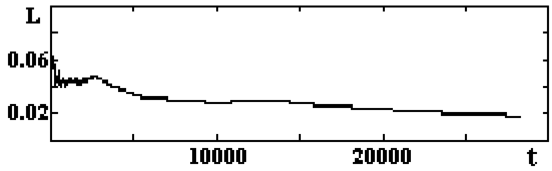

Fig. 4. Time dependence of Lyapunov index.

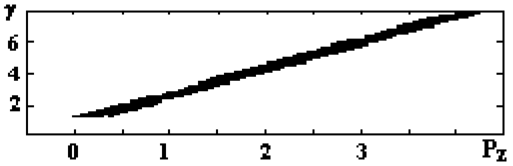

Fig. 5. Time dependence of particle energy.

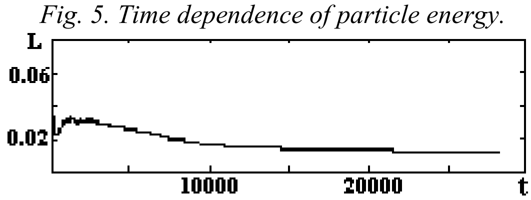

Fig. 6. Time dependence of Lyapunov index

From these figures one can see, that at the chosen parameters of problem the particle dynamics with initial phase ψ =0 is chaotic. Particles with other initial phases behave similarly.

On Fig. 7-8 the time dependences of average energy $<\gamma>$ and $S^2 = <(\gamma - \gamma_o)^2>$ (average on ensemble) are represented at particles motion in the field of wave with circular polarization (a) and in the field of E-wave (b).

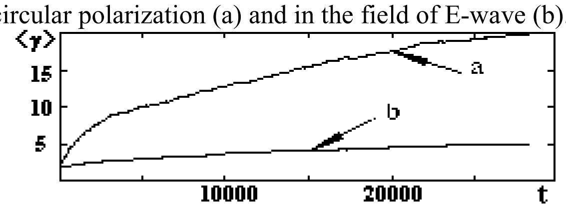

Fig. 7. Time dependence $< \gamma >$.

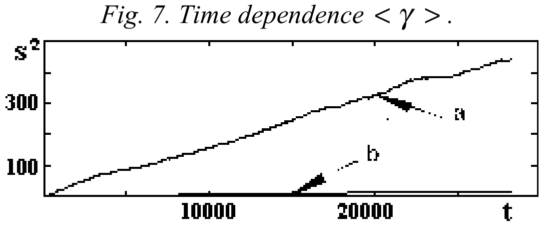

Fig. 8. Time dependence $S^2 = <(\gamma - \gamma_o)^2>$.

It is seen that the rate of energy gain by particles in case (a) considerably exceeds the one in case (b). In case (b), average energy is $\langle\gamma\rangle \propto 5$. The close value for average energy gives criterion (19) $\gamma_{16} \propto 6$. The diffusion coefficient, in case (a) more than on the order exceeds diffusion coefficient obtained in case (b).

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