BUNCHING SYSTEM BASED ON THE EVANESCENT WAVES

M.I. Ayzatsky, K.Yu. Kramarenko, S.A. Perezhogin
National Science Center Kharkov Institute of Physics&Technology
1 Academicheskaya, NSC KIPT, 61108 Kharkov, Ukraine
E-mail aizatsky@nik.kharkov.ua

To improve the beam bunching at the initial stage of acceleration it is necessary to create an increasing field distribution. Such distribution can be created in the ordinary disk-loaded waveguide in its stopbands. Generally, there are two eigen evanescent waves, one of which has increasing distribution along the longitudinal axes and another - decreasing one. The field structure in the bounded system is a superposition of these two evanescent waves whose amplitudes are determined by the operating frequency and by the geometry of the boundary cavities. The results of the simulation of the buncher for the case of the 25 keV injected electron energy are presented in the paper.
PACS numbers: 29.27.-a

1 INTRODUCTION

To improve the bunching process at the initial stage of acceleration it is necessary to create an increasing field distribution. It was shown, that the amplitude distribution of the eigen oscillations in the bounded periodic structure within the stopband corresponds to the increasing amplitude distribution [1]. It is known, that in the boundless periodic structure two eigen electromagnetic oscillations exist. In the passbands the eigen oscillations represent travelling waves. In the stopbands the eigen oscillations do not transfer energy in the direction of periodicity and have either a decreasing or increasing character. In the bounded periodic structure it is possible to create the field distribution corresponding to the one (increasing or decreasing) eigen oscillation. The increasing field distribution cannot be obtained in the smooth waveguide. As a result of the boundary conditions, the amplitude of the increasing solution in the smooth waveguide is always less than the amplitude of the decreasing one.

2 MATHEMATICAL MODEL

Bunching system based on the segment of cylindrical disk-loaded waveguide is considered. To investigate the field distribution in this structure we use the oscillation equations of the weakly coupled cavities. The coupling between the neighboring cavities is taken into account [2].

The most of the accelerating structures operate in the E_{010} – mode. The field amplitude distribution of E_{010}– mode in the structure consisting of N cavities is determined by the set of N equations:

$$A_1\left(\omega^2-\omega_1^2(1+\varepsilon_1)-\frac{i\omega\omega_1}{Q_1}\right)+\omega_1^2\tilde{\varepsilon}_1 A_2=0, \qquad (1.1)$$

$$A_2\left(\omega^2-\omega_0^2(1+2\varepsilon)-\frac{i\omega\omega_0}{Q_0}\right)+\omega_0^2\varepsilon A_3+\omega_0^2\tilde{\varepsilon}_1 A_1=0, \quad (1.2)$$

$$A_n\left(\omega^2-\omega_0^2(1+2\varepsilon)-\frac{i\omega\omega_0}{Q_0}\right)+\omega_0^2\varepsilon A_{n-1}+\omega_0^2\varepsilon A_{n+1}=0, \quad (1.3)$$

$$A_{N-1}\left(\omega^2-\omega_0^2(1+2\varepsilon)-\frac{i\omega\omega_0}{Q_0}\right)+\omega_0^2\varepsilon A_{N-2}+\omega_0^2\tilde{\varepsilon}_N A_N=0, \quad (1.4)$$

$$A_N\left(\omega^2-\omega_N^2(1+\varepsilon_N)-\frac{i\omega\omega_N}{Q_N}\right)+\omega_N^2\tilde{\varepsilon}_N A_{N-1}=0, \quad (1.5)$$

where A_n is the field amplitude in n-th resonator, $n=1, 2, \ldots N$; $\chi=\frac{2}{3\pi J_1^2(a_1)}$, $\varepsilon=\chi\frac{a^3}{b^2 D}$, $\varepsilon_{1,N}=\chi\frac{a^3}{b_{1,N}^2 D_{1,N}}$, $\tilde{\varepsilon}_{1,N}=\chi\frac{a^3}{bb_{1,N}\sqrt{DD_{1,N}}}$, a is the radius of coupling aperture, b is the radius of the cavity, D is the length of the cavity, ω_0 is the resonant frequency of the n-th cavity ($n\neq 1,N$), ω_1 is the resonant frequency of the 1-st cavity, ω_N is the resonant frequency of the N-th cavity. Equations (1.1-1.5) are true in the case of thin diaphragms. The set of equations (1.1-1.5) is the set of homogeneous difference equations of the second order with constant coefficients. The field amplitude distribution in the boundless structure is determined by the set of equivalent equations, which are similar to the equation (1.3). There are two partial solutions of this equation: $A_n=\rho_1^n$ and $A_n=\rho_2^n$, where $\rho_{1,2}=\beta\pm\sqrt{\beta^2-1}$ are the roots of the characteristic equation $\rho^2-2\beta\rho+1=0$. The value β is defined as:

$$\beta=\left(\omega_0^2(1+2\varepsilon)-\omega^2+\frac{i\omega\omega_0}{Q_0}\right)/2\omega_0^2\varepsilon. \qquad (2)$$

At first, the ideal structure ($Q_0=\infty$) will be considered. In the passband, when the frequency changes from ω_0 ($\beta=1$) up to ω_π ($\beta=-1$), $\rho_1=e^{i\psi n}$ and $\rho_1=e^{-i\psi n}$. The phase shift per cell is defined from the dispersion equation: $\cos\psi=\beta$. In the stopbands, $\omega<\omega_0$ and $\omega>\omega_\pi$, the values ρ_1, ρ_2 correspond to the evanescent oscillation: $|\rho_{1,2}|\neq 1$. At the frequencies $\omega>\omega_\pi$ ρ_1,ρ_2 take the negative values: $\rho_1=|\rho_1|e^{i\pi}$, $\rho_2=|\rho_2|e^{i\pi}$.

The field amplitude distribution in the bounded structure equals to the sum of two partial solutions with constant coefficients – $A_n=C_1\rho_1^n+C_2\rho_2^n$. Excluding A_1 from (1.1, 1.2) and A_N from (1.4, 1.5), we get the set of equations to define C1, C2:

$$C_1\rho_1^2\left(f_1(\omega)+\rho_1\right)+C_2\rho_2^2\left(f_1(\omega)+\rho_2\right)=0\,, \qquad (3.1)$$

$$C_1\rho_1^{N-1}\left(f_N(\omega)+\rho_2\right)+C_2\rho_2^{N-1}\left(f_N(\omega)+\rho_1\right)=0\,, \quad (3.2)$$

where $f_1(\omega)=-2\beta+\omega_1^2\tilde{\varepsilon}_1^2/\varepsilon\left(\omega_1^2(1+\varepsilon_1)-\omega^2\right)$,

$f_N(\omega)=-2\beta+\omega_N^2\tilde{\varepsilon}_N^2/\varepsilon\left(\omega_N^2(1+\varepsilon_N)-\omega^2\right)$.

The set of equations (3) has the non-trivial solution if determinant is equal to zero:

$$\begin{aligned}&\rho_1^2\left(f_1(\omega)+\rho_1\right)\rho_2^{N-1}\left(f_N(\omega)+\rho_1\right)-\\&-\rho_2^2\left(f_1(\omega)+\rho_2\right)\rho_1^{N-1}\left(f_N(\omega)+\rho_2\right)=0.\end{aligned} \qquad (4)$$

This equation determines the resonant frequencies of the bounded structure. The structure consisting of N cavities has N resonant frequencies. Resonant frequencies of the structure with half boundary cells ($D_1 = D_N = D/2$) lay within the interval $\omega_0 \le \omega_n \le \omega_\pi$. In this case, the resonant oscillation represents a standing wave formed by two traveling waves – $e^{i\psi_n n}$ and $e^{-i\psi_n n}$, $\psi_n = \pi n/(N-1)$, $n=0, \dots N-1$.

Let us consider the case, when the cavity chain has arbitrary (not half) boundary cells. Suppose, that at the some frequency ω_1' the following conditions are fulfilled:

$$f_1(\omega_1')+\rho_2=0\,,\ f_1(\omega_1')+\rho_1\neq 0\,, \qquad (5.1)$$

$$f_N(\omega_1')+\rho_1=0\,,\ f_N(\omega_1')+\rho_2\neq 0\,. \qquad (5.2)$$

If boundary cells differ from other ones only in length - $D_1 = \xi_1 D$, $D_N = \xi_N D$, than conditions (5.1), (5.2) will have the form:

$$\xi_1=\frac{1}{1-\rho_1}\,, \qquad (6.1)$$

$$\xi_N=\frac{1}{1-\rho_2}\,. \qquad (6.2)$$

In this case, as it follows from equations (3.1, 3.2), $C_1=0$. The field amplitude distribution is as follows: $A_n=C_2\rho_2^n$.

By similar way, one can obtain conditions for creating the field amplitude distribution of the form $A_n=C_1\rho_1^n$ at the some frequency ω_2'.

At the frequency ω_1' (ω_2') the resonant oscillation of the structure consisting from N cavities is based on the one eigen oscillation of the boundless structure. As ξ_1, $\xi_N > 0$, than $\omega_1', \omega_2' > \omega_\pi$. Hence, the eigen oscillation is based on evanescent oscillation of the boundless structure. Let us suppose, that at the frequency $\omega > \omega_\pi$ $|\rho_1|$ is less than one ($|\rho_1|<1$) and $|\rho_2|$ is grater than one ($|\rho_2|>1$). So, at the frequency ω_1' the amplitude distribution in the structure ($A_n=C_2\rho_2^n$) is an increasing one: $|A_{n+1}|>|A_n|$; at the frequency ω_2' the amplitude distribution in the structure ($A_n=C_1\rho_1^n$) is a decreasing one: $|A_{n+1}|<|A_n|$. It is easy to show that both for increasing and decreasing amplitude distributions the following condition is satisfied:

$$D_1 + D_N = D. \qquad (7)$$

The resonant oscillation of the structure with $D_1 > D/2$, $D_N < D/2$, $D_1 + D_N = D$ at the frequency $\omega > \omega_\pi$ corresponds to the increasing eigen oscillation (Fig. 1, curve b); as for the decreasing one, the length of the boundary cavities must fulfill such conditions: $D_1 < D/2$, $D_N > D/2$, $D_1 + D_N = D$ (Fig. 1, curve a).

Consider the case, when the structure is exited at the frequency ω_1'. Taking into account losses in the structure, one can obtain:

$$\frac{C_1}{C_2}\approx\rho_2^{2(N-1)}\delta(Q)\,. \qquad (8)$$

As one can see, the amplitude distribution depends on the number of cavities. When $\rho_2^{2(N-1)}\delta(Q) << 1$, than $C_1 << C_2$. In this case, the amplitude distribution in the structure is increasing. If $\rho_2^{2(N-1)}\delta(Q)\approx 1$, then the amplitude distribution represents a superposition of two eigen oscillations. When $N \to \infty$, than $C_1 >> C_2$ and the amplitude distribution is decreasing. It satisfies the condition that in the half-bounded structure the increasing distribution cannot be realized.

One resonant frequency of the structure, the boundary cells of which differ from half ones, lays outside the passband: $\omega > \omega_\pi$. If condition (7) is not fulfilled, the resonant oscillation represents the sum of two evanescent oscillations (Fig. 1, curve c).

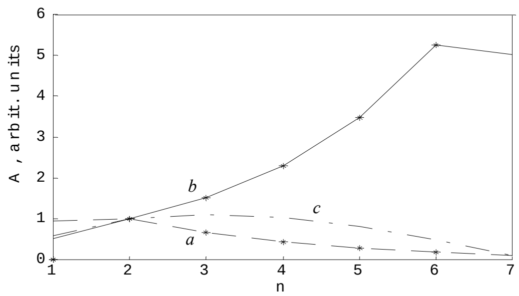

Fig. 1. Amplitude distribution in the structure consisting of 7 cavities at the frequency $\omega=1.004\omega_\pi$: a=1.5 см, b=4.4026 см, D =2.438 см; curve a – D_1 =0.9698 cm, D_N = 1.4682 cm; curve b – D_1 =1.4682 cm, D_N= 0.9698 cm; curve c – D_1 =1.4682 cm, D_N =1.219 cm.

We considered the situation, when we changed only the length of the boundary cells. Similar effect can be achieved by changing the eigen frequencies of boundary cavities. Resonant frequency of the E_{010} - mode is in inverse proportion to the radius of the cavity. Let us designate – $b_1 = \zeta_1 b$, $b_N = \zeta_N b$. We shall suppose that $D_1 = D_N = D$.

For the realization of the increasing field amplitude distribution $A_n=C_2\rho_2^n$, $C_1 = 0$ (Fig. 2, curve b) it is necessary to select the resonant frequency of the bounded structure laying above ω_π, and to satisfy conditions (5.1, 5.2). Setting $\omega_1' > \omega_\pi$, we define the values ρ_1, ρ_2. Coefficients ζ_1, ζ_N, which determine the radii of boundary cavities with respect to other ones, are defined from two nonlinear equations:

$$\rho_1=\frac{1}{2\zeta_1^4}\left[(\alpha_1+\gamma_1)-\sqrt{(\alpha_1+\gamma_1)^2+4\zeta_1^4\left(1-\zeta_1^4\right)}\right], \quad (9.1)$$

$$\rho_2=\frac{1}{2\zeta_N^4}\left[(\alpha_N+\gamma_N)-\sqrt{(\alpha_N+\gamma_N)^2+4\zeta_N^4\left(1-\zeta_N^4\right)}\right]. \quad (9.2)$$

To obtain the decreasing amplitude distribution one can reverse boundary cells (Fig. 2, curve a).

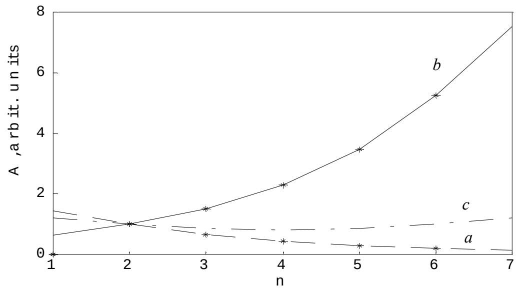

Fig. 2. Amplitude distribution in the structure consisting of 7 cavities at the frequency $\omega=1.004\omega_\pi$: a=1.5 cm, b=4.4026 cm, D =2.438 cm; curve a – b_1 =4.1636 cm, b_N = 4.2516 cm; curve b – b_1 =4.2516 cm, b_N = 4.1636 cm; curve c – b_1 = b_N =4.1825 cm.

3 SIMULATION

Proceeding from the above-presented theory, the injector system based on evanescent oscillation was simulated using SUPERFISH [3] and PARMELA [4] codes. The simulation was held under the electron beam initial energy W_0=25 keV and current 50 mA with space charge forces taken into account. Peak value of on-axis electric field is 30 MV/m.

Waveguide section composed from five accelerating cells was taken for simulations of bunching system based on disk-loaded waveguide. It is well known that the disk-loaded waveguide has many stopbands. As a working stopband we have chosen the second stopband of the symmetric wave. If we want to work in the stopband, the conditions for the eigen frequency of the system to lay in the stopband must be created. The simplest way of creating such situation is shifting the frequency of the last cell. In the second stopband the phase shift per cell equals π (ρ<0). The time-transit angle for relativistic particle was chosen equal to 0.3π per period. As a result of simulations, the on-axis increasing field distribution was obtained. The results are shown in Fig. 3.

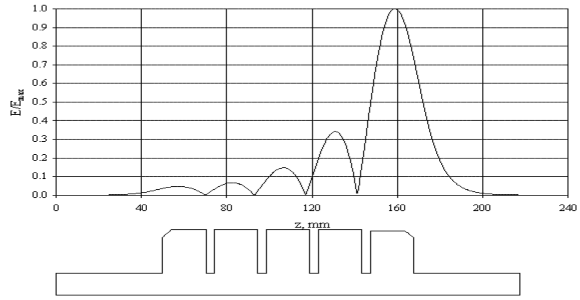

Fig. 3. Geometry of bunching system base on disk-loaded waveguide and corresponding on-axis electric field distribution.

Electrodynamic performances of the simulated system are the follows: the quality factor Q=11163, shunt impedance R_{sh}=39.8 MOhm/m. Simulation of particle dynamics in the system has shown that the maximum energy is 0.779 MeV, average energy is 0.697 MeV, energy spectrum is 9% (70% of particle), phase length is 32°, normalize emittance is 28 mm·mrad and capture is 91.2%.

4 CONCLUSION

The increasing amplitude distribution necessary for the effective bunching process can be obtained in the regular disk-loaded waveguide. Results of simulation of the bunching process show the efficiency of using the bunching system on evanescent oscillations. It was also shown that in the bounded structure with losses amplitude distribution depends on the number of resonators.

The authors express gratitude to V.A. Kushnir and V.V. Mitrochenko for the participation in discussing the results.

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