USING OF UNDULATORS FOR LOW ENERGY ION LINAC

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The possibility of using undulators for focusing and acceleration of charged particles in RF field is discussed. There are suggested that the RF field does not have harmonics in synchronism with the beam. The accelerating force is produced by a combination of RF and undulator fields. Examples illustrating the efficiency of the proposed method acceleration are given for low energy ion beams. In the undulator accelerator (UNDULAC) an electrostatic, a magnetic and radio frequency undulators can be used. The focusing conditions of the beam are studied. Methods for increasing of the ion beam intensity are discussed.
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1 INTRODUCTION

In a conventional RF linac the beam is accelerated by a synchronous wave. Another method to accelerate ions – in the fields without a synchronous wave – was suggested in Ref. [1] in which case the accelerating force is to be driven by a combination of two non-synchronous waves (two undulators). In undulator linac in question, one of the undulators must be of the RF type (it drives non-synchronous RF wave field), the second one being, optionally, of magnetic, electrostatic, or radio frequency types. The 3D dynamics of the ion beam in undulator linear accelerator (UNDULAC) will be determined by the type of undulator and transverse structure of its field. In this paper a theoretical aspects of the method of ion acceleration and focusing in the fields without synchronous wave and possible realization undulator accelerator for ion are investigated. The results are compared with a conventional ion RF linac.

2 PARTICLE MOTION EQUATION

The motion equation of a particle in the field of two waves can be written using Lagrangian function, as

$$\frac{d\mathbf{P}}{dt} = e\nabla(\mathbf{v}\cdot\mathbf{A}_\Sigma - \Phi_\Sigma), \quad (1)$$

where $\mathbf{P} = \mathbf{p} + e\mathbf{A}$ is the generalized momentum, $\Phi_\Sigma(\mathbf{r})$ is the electrostatic potential, $\mathbf{A}_\Sigma(\mathbf{r}) = \mathrm{Re}[\mathbf{A}_n(\mathbf{r}^\perp)e^{i\varphi_n} + \mathbf{A}_l(\mathbf{r}^\perp)e^{i\varphi_l}]$ is the total vector potential of the two fields with amplitudes $\mathbf{A}_{n,l}$, phases $\varphi_{n,l} = \int k_{n,l}\,\mathrm{d}z - \omega_{n,l}t + \alpha_{n,l}$ and wave numbers $k_{n,l}$. When the phase velocities $v_{ph,n,l} = \omega_{n,l}/k_{n,l}$ differ significantly from the average velocity of the particles v_b, the trajectories of the individual particles of the beam in general have a complicated shape but can always be represented as the sum of a slow variation $\bar{\mathbf{r}}$ and rapid oscillation $\tilde{\mathbf{r}}$. Accordingly, the beam momentum $\mathbf{p}$ can be represented as the sum of a slowly varying and a rapidly oscillating component, $\mathbf{p} = \bar{\mathbf{p}} + \tilde{\mathbf{p}}$. After an average is taken over the rapid oscillations from (1) we can obtain the time-averaged equation of nonrelativistic ion motion if the beam velocity $v_b \approx v_c \equiv (\omega_n \pm \omega_l)/(\mathrm{k}_n \pm \mathrm{k}_l)$:

$$\frac{d^2\bar{\mathbf{r}}}{dt^2} = -\frac{d}{d\bar{\mathbf{r}}}U_{eff}, \quad (2)$$

where 3D effective potential

$$U_{eff} = U_1(\bar{\mathbf{r}}^\perp) + U_2(\bar{\mathbf{r}}^\perp, \psi), \quad (3)$$

$$U_1 = \frac{e^2}{4m^2}\left(|\mathbf{A}_n|^2 + |\mathbf{A}_l|^2\right), \quad (4)$$

$$U_2 = \frac{e^2}{2m^2}\mathrm{Re}\left(\mathbf{A}_n\cdot\mathbf{A}_l^{*}e^{i\psi}\right), \quad (5)$$

$\bar{\mathbf{r}}^\perp$ and $\psi = (k_n \pm k_l)\bar{z} - (\omega_n \pm \omega_l)t + (\alpha_n \pm \alpha_l)$ are slowly varying transverse coordinate and phase.

We see from (3) that the longitudinal bunching and accelerations of the beam are provided by a combined wave with phase velocity $\mathrm{v_c}$ which is close to the average particle velocity. The choice of the functions $\mathbf{A}_n(\bar{\mathbf{r}}^\perp)$ and $\mathbf{A}_l(\bar{\mathbf{r}}^\perp)$ is not arbitrary because simultaneously to acceleration it is necessary to keep up the transverse focusing of the beam. Equilibrium trajectories can exist for all particle phases if the following conditions hold in the injection plane:

$$\frac{\partial}{\partial\bar{\mathbf{r}}^\perp}U_1 = 0, \qquad \frac{\partial}{\partial\bar{\mathbf{r}}^\perp}U_2 = 0. \quad (6)$$

Conditions (6) are necessary but not sufficient conditions for focusing of the beam. In the absence of resonance there can be transverse stability only if the «effective potential» U_{eff} has a minimum in the transverse plane (X,Y) This imposes a limitation on the amplitudes $\mathbf{A}_n(\bar{\mathbf{r}}^\perp)$, $\mathbf{A}_l(\bar{\mathbf{r}}^\perp)$ and confine configuration of the fields. The necessary condition of simultaneous transverse and longitudinal focusing is the existence of an absolute minimum for U_{eff}. For low energy ion accelerator it is difficult to create resonator system with two generators, when $\omega_n \neq \omega_l$. It is interesting to consider some versions of the linear undulator accelerator:

1. UNDULAC-E(M) that employs a combination of RF field ($\omega_n = 2\pi c/\lambda$, $k_n = 2\pi c/\beta_{ph,n}\lambda$) and static periodical electric (or magnetic) field of an undulator ($\omega_l = 0$, $k_l = \mu_0/\lambda_0 + 2\pi l/\lambda$), where l=0,1,2...; λ_0 is slowly varying period of structure, μ_0 is a phase advance of field per period;
2. UNDULAC-RF that employs a combination of two

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space nonsynchronous harmonics of RF field in the periodical resonator structure ($\omega_n = \omega_l = 2\pi c/\lambda$, $k_n \neq k_l$), where $k_n = \mu_v/\lambda_0 + 2\pi n/\lambda_0$, μ_v is phase advance per period of RF structure, *l*, *n*=0,1,2,... and *n*≠*l*.

3 ION BEAM FOCUSING AND ACCELERATION IN UNDULAC

In UNDULAC-E (M) the rate of acceleration is proportional to the amplitudes of the RF field E_v and undulator fields E_o (B_o). Still, increase of the beam energy occurs due to RF field only. As is shown in [2], the energy gain is found by $dW_c/dz = eT_{e,m}E_v \cos\psi_c$ where $T_{e,m}$ are the acceleration efficiency factors for UNDULAC-E and UNDULAC-M. The choice of RF field amplitude E_v and undulator field amplitude E_o (B_o) is not independent, since it is necessary to realize beam focusing and to provide a large transmission coefficient *K*. For example, if the RF field and the undulator electric field have only single space harmonics ($n = 0,\ l = 0$); phase advance per period of structure $\mu_v = 0$ for RF field and $\mu_0 = \pi$ for undulator, focusing and acceleration occur for all paraxial particles, when $\beta_c = 2\lambda_0/\lambda$ and $\chi = E_0/E_v \geq 2/3$. In this case the acceleration efficiency factors $T_e = \chi\dfrac{eE_v\lambda}{4\pi mc^2\beta_s}$. For a plane magnetic undulator the function U_{eff} has minimum, and the focusing takes place for all phase ψ if $B_v = \beta_c B_0$. This condition connects values B_v, B_0 and β_c and permits to express the factor T_m through the amplitude of RF field, $T_m = \dfrac{eE_v\lambda}{4\pi mc^2\beta_c}$. This value is close by T_e.

The combined acceleration field can be driven without use of a magnetic or an electrostatic undulator. Indeed, consider ion beam dynamics in the periodical RF structure without synchronous space harmonics. Interaction of the beam with each harmonic can be treated as ion interaction with a radio frequency undulator (UNDULAC-RF). The combined field of two harmonics would accelerate the beam if $\beta_b \approx \beta_c = \omega/ck_c$ where k_c is a wave number of a combined wave field, $k_c = (k_n \pm k_p)/2$ ($k_c \neq k_n \neq k_p$, $n = 0,1,2,...$, $p = 0,1,2,...$). The rate of energy gain $dW_c/dz = eT_{rf}E_{v,1}\cdot\sin 2\psi$. The acceleration and phase stability of the beam are possible when the phase of a synchronous particles ψ_c in the combined field is in an intervals $[\pi/4, \pi/2]$ and $[5\pi/4, 3\pi/2]$. In this case the frequency of ion beam bunching is double ($\omega_b = 2\omega$).

For undulator, where RF field has a phase advance μ=0 per a period, and only two harmonics n=0, p=1 are taken into acount, the transverse focusing of the beam is possible if the amplitude of the first harmonic is larger than zero one ($E_1 > E_0$) and the beam velocity $\beta_b \approx \beta_c = 2\lambda_0/\lambda$. For this case the acceleration efficiency factor is $T_{rf} = \dfrac{eE_{v,0}\lambda}{4\pi mc^2\beta_s}$. The rate of energy gain is the same as in UNDULAC-E(M).

For the undulator, where the RF field has a phase advance per period $\mu_v = \pi$ and beam velocity $\beta_b \approx \beta_c = \lambda_0/\lambda$, value of the acceleration efficiency factor $T_{rf} = \dfrac{eE_{v,0}\lambda}{2\pi mc^2\beta_s}$. Besides, the choice of harmonic amplitudes E_0 and E_1 are independent because the condition of focusing in UNDULAC-RF can be fulfilled for any value of E_0/E_1. The maximum values of E_0 and E_1 can be found from RF characteristic of the resonator, the transverse acceptance and beam current.

4 UNDULAC AND RF FOCUSING IN LINAC

It is interesting to compare the methods of acceleration in UNDULAC and in linac under consideration with axisymmetric RF focusing (ARF). The main principles of APF can be described by means of two wave approximation method: in a periodical RF structure the beam is accelerated by a synchronous space harmonic of the wave while another, nonsynchronous, harmonic is only responsible for focusing the particles. By means of Hamiltonian analysis in the 4-dimensional phase space it is possible to find the relationship between the defined longitudinal acceptance and the limit value of transverse beam emittance, which provides the maximal transmission coefficient *K*≈1 [3]. For two- wave approach the rate of energy gain $dW_c/dz = eT_sE_{v,n}\cos\psi_c$. The beam focusing takes place and *K*≈1 if the amplitude of nonsynchronous harmonic $E_n >> E_s$. For two fundamental harmonics $s = 0,\ n = 1$ and a phase advance per period $\mu_v = \pi$, the efficiency factor for axisymmetric RF focusing $T_s \approx \dfrac{eE_n\lambda}{4\pi mc^2\beta_s}$. This magnitude is similar to. $T_{e,m}$, but it is twice as low than T_{rf}.

The using of undulators for acceleration and focusing of ion beams is extremely promising. First, the problem of design of UNDULAC RF system is simplified considerably, since focusing and acceleration of particles is possible for both transverse (TE or TEM) and longitudinal (TM) RF fields without any external focusing elements and dedicated slow-wave systems. No drift tubes are required for TEM wave. Second, an efficient bunching and a large transmission coefficient of particles can be achieved solely by changing the amplitude and period of the static undulator field. This eliminates the serious problems involving adjustment and matching of the RF system since the latter can be made uniform. Third, an UNDULAC can be used for acceleration high intensity ion beams [4]. Indeed, the main factor limiting beam intensity in ion accelerator is space charge forces. There exist, at least, three way to increase ion beam intensity in a linear undulator accelerator: (i) to enlarge beam cross-section; (ii) to accelerate several beams in a channel of RF structure; (iii) to compensate for the space charge by accelerating ions with opposite charge signs within the same bunch.

(i) In a UNDULAC where there are no drift tubes, a ribbon or a hollow beams having large cross-sections can be accelerated. Acceleration of a ribbon ion beam with the current J>1A in a plane electrostatic undulator was studied in [5]. It was shown that a large cross-section and electrostatic shielding of the space charge field decreases Coulomb defocusing of the particles in the narrow accelerating channel.

(ii) In the new accelerator one can accelerate the several beams in a single channel of RF structure since there are no drift tubes involved. The problem is to choose a dedicated symmetry of the transverse radio-frequency and periodic magnetic field. The RF system must be a small transverse size. It is preferable to use a shielded multielectrode line where transverse electromagnetic waves (TEM) can propagate. Configuration of the RF field and magnetic undulator field must be such as to maintain several equilibrium trajectories simultaneously [4].

(iii) Study of feasibility of simultaneous acceleration of both positive and negative ions with identical charge-to-mass ratio within the same bunch is of great interest. The current limit of the ion beam can be increased significantly by using the space charge compensation of positively H^+ (D^+) and negatively H^- (D^-) charged ions accelerated in the same bunch. This conclusion can be drawn from Eq.(1). Indeed, the effective potential U_{eff} depends on the particle charge squared, i.e. averaged motions of positive and negative charged ions are identical. It allows to increase the beam current limit.

All possible methods of focusing and acceleration in undulator linear accelerator would be effective for low energy ion, when $\beta_b \cong \frac{eE_n\lambda}{4\pi mc^2}$. The acceleration efficiency factor $T_{m,e,rf}$ decreases with growth of the beam velocity like in the RFQ accelerator. Therefore UNDULAC can be used as an initial part of the high intensity linear accelerator (buncher) or as aninjector for the neutron generators and nuclear fusion reactor.

5 CONCLUSION

Theoretical studies of using undulators in the acceleration system showed a possibility to create a new type of ion linear accelerator (UNDULAC). Three types of undulator for ion acceleration linac are suggested. The requirement of particle focusing imposes a limitation on the field amplitudes in undulators of all types besides UNDULAC-RF with $\mu_v = \pi$, where the focusing condition holds for any relation between harmonic amplitudes. The rate of energy gain in UNDULAC is comparable with analogous value for RFQ and conventional linac, where RF focusing is realized. But the new accelerator has a number of advantages. In this accelerators it is possible to use not only a longitudinal (TM) but also a transverse (TE or TEM) radio frequency field. For the TEM wave the drift tubes are absent. The beam intensity can be increased in these RF structures by means of construction of a multibeam channel or enlarging the beam cross-section. The other important way to increase the beam intensity is using the space charge compensation. In UNDULAC positive and negative ions would be within the same bunch and the current limit of the ion beam can be substantially increased.

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