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ON THE SYNCHROTRON RADIATION OF ULTRARELATIVISTIC ELECTRONS MOVING ALONG CURVED SPIRAL TRAJECTORY

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The synchrotron radiation of an ultrarelativistic charged particle moving along spiral trajectory winded on curved magnetic force line is considered. The radiation pattern has new properties on a comparison with the radiation in homogeneous magnetic field: there is a range of characteristic frequencies instead of one characteristic frequency, the peaks of the radiation pattern correspond to periodically repeated directions in space, which position depends on the frequency of radiation.

1. INTRODUCTION

The formulae for the synchrotron radiation mechanism in a homogeneous magnetic field [1-3] are widely applied in various branches of science and engineering [4]. However, the formulae for synchrotron radiation in straight magnetic field lines may be insufficient to describe radiation of ultrarelativistic electrons moving along dipolar field lines in the magnetosphere of a pulsar [5]; or the radiation emitted by runaway electrons in tokamaks [6]. It is necessary to take into account the curvature of the magnetic force line [5, 6].

Since synchrotron radiation comes from a small length along the trajectory, the curved magnetic field lines are approximated by circular force lines and the radiation from relativistic electrons moving with small pitch angles along spiral trajectory is considered. The radiation formulae have been calculated by various methods in the papers [5 - 8]. In [5] the radiation was called as synchrotron curvature. In [5, 7] the spectrum and polarization characteristics of radiation was obtained, in [6] the radiation spectrum was found. Expressions for the spectrum obtained in the papers [5 - 7] have various forms. The comparison of the formulae [5] and [7] is carried out in [8]. The limit of an undulator radiation, when the contribution to radiation occurs from a lot of cyclotron rotations, has been considered in [9, 10].

At the same time the spectral angular distribution of synchrotron radiation emitted by ultrarelativistic charged particles moving along curved spiral trajectory is not investigated. In the present paper such spectral angular distribution is studied and the comparison of expressions for the radiation spectrum obtained in the papers [6, 7] will be also carried out.

2. TRAJECTORY OF PARTICLE

Let us assume that magnetic force lines look like a circle, and the magnitude of magnetic field is B_0. Select a system of Cartesian coordinates with (x, y)-axes in the plane of magnetic field lines, and z-axis coinciding with the axis of cylindrical magnetic surface. The magnetic field vector can be expressed as

$$\mathbf{B} = B_0(\sin\varphi\,\mathbf{i} - \cos\varphi\,\mathbf{j}), \quad (1)$$

where φ is the polar angle in (x, y)-plane, **i**,**j** are the basis vectors of Cartesian frame. The particle with the Lorentz-factor $\gamma = (1 - v^2/c^2)^{-1/2} >> 1$ is moving along magnetic force lines with the velocity close to speed of light. The angular velocity Ω corresponding this motion ($\Omega \equiv v_\parallel / R$, where $v_\parallel$ is the velocity of the guiding center along the magnetic line with curvature radius R) is much less than the frequency of rotation around magnetic force line ω_B, $\Omega << |\omega_B|$. The radius of Larmor circle r_B is much less than R, $r_B << R$.

Equations of motion of a charged particle in the magnetic field (1) are integrated in quadratures. The solution expresses through elliptic integrals of the 1-st and 3-rd kind. The asymptotic expansion of the position vector of trajectory, in which the terms proportional $(r_B / R)^2 << 1$ are dropped, has the form [7, 8]

$$\begin{aligned}\mathbf{r} &= \left[-2\delta r_B \sin\omega_B t \cos\Omega t + (R + r_B\cos\omega_B t)\sin\Omega t\right]\mathbf{i} + \\ &+ \left[2\delta r_B \sin\omega_B t \sin\Omega t + \left(R + r_B\cos\omega_B t\right)\cos\Omega t\right]\mathbf{j} + \\ &+ \left[v_D t - r_B\sin\omega_B t\right]\mathbf{k},\end{aligned} \quad (2)$$

where $\omega_B = e_\alpha B_0 / m_\alpha c\gamma$, $\delta = \Omega/\omega_B << 1$, $v_D = -\Omega^2 R/\omega_B$ is the drift velocity, e_α and m_α is the charge and mass of a particle of a sort α, **i**, **j**, **k** are the basis vectors of the Cartesian frame.

In contrast to known expressions of drift theory, the terms, which are proportional $2\delta\sin\omega_B t$, is taken into account. It is necessary to reduce evaluations in the Cartesian frame to evaluations in the frame of natural trihedral [7, 8].

The magnitude of particle velocity remains constant and is given by expression

$$v = (\Omega^2 R^2 + v_D^2 + \omega_B^2 r_B^2)^{1/2}. \quad (3)$$

The curvature radius of trajectory (2) is equal to

$$r_C = k^{-1} = v^2 / \Omega^2 R(1 + q^2 + 2q\cos\omega_B t)^{1/2}, \quad (4)$$

where k is the curvature, the parameter $q = \omega_B^2 r_B / (\Omega^2 R)$ is equal to the ratio of the Larmor velocity $v_L = |\omega_B| r_B$ to the magnitude of drift velocity.

Further we shall consider the case, for which the projection of a particle velocity on magnetic lines is close to speed of light, $v_\parallel \to c$. Thus the Lorentz-factor corresponding to motion along magnetic field lines, $\gamma_\parallel = (1 - v_\parallel^2 / c^2)^{-1/2} >> 1$.

If the magnitude of magnetic field depends on radial coordinate $B_0 \to B(r)$, the corresponding drift velocity $v_g = (1/2)v_\perp^2 / (\omega_B R)$, (where $v_\perp$ is the velocity transverse to magnetic field lines) is smaller than the velocity of centrifugal drift $v_D = (1/2)v_\parallel^2/(\omega_B R)$. Therefore, we shall consider the constant magnetic field approximation.

It is known that radiation of a relativistic charged particle occurs from the small part of trajectory and concentrates within the angle $\sim 1/\gamma$ at apex of cone along particle's velocity [4, 11]. Thus, the instantaneous angle $\sim 1/\gamma$ of the radiation beam should be less than the angle between the particle velocity and drift trajectory. From this requirement, definition of Lorentz-factor γ, and inequality $\gamma_\parallel >> 1$ follows that the limit of synchrotron radiation takes place, if

$$\gamma^2 >> \gamma_\parallel^2 \tag{5}$$

Suppose that the inequality (5) is fulfilled.

3. SPECTRAL ANGULAR DISTRIBUTION OF SYNCHROTRON RADIATION

The energy E emitted by a charged particle in the solid angle between o and $o+do$, and the interval of frequencies between γ and $\omega + d\omega$ is given by [11]

$$dE = \frac{cR_0^2}{4\pi^2}\left|\mathbf{E}(\omega)\right|^2 do\, d\omega, \tag{6}$$

where the Fourier integral representation of an electrical field is

$$\mathbf{E}(\omega) = \frac{-i\omega e_\alpha}{cR_0} e^{\{\frac{i\omega R_0}{c}\}} \int_{-\infty}^{+\infty} [\mathbf{n}[\mathbf{n},\boldsymbol{\beta}]]\exp\{i\omega(t - \mathbf{nr}/c)\}dt. \tag{7}$$

Here R_0 is the distance up to the observer, $\mathbf{n}$ is the unit vector pointing to the observer, $\boldsymbol{\beta} = \mathbf{v}/c$, $\mathbf{v}$ is the velocity, $\mathbf{r}$ is the particle position vector (2).

To calculate integral (7), we use a frame of natural trihedral at time t_0. Denote by $\boldsymbol{\tau} = \mathbf{v}(t_0)/v$, $\boldsymbol{\nu}(t_0)$, $\mathbf{b}(t_0)$ the tangent, normal, and binorma, respectively. By definition, the instant t_0 is found from requirement that the vector $\mathbf{n}$ belongs to ($\boldsymbol{\tau}$, $\mathbf{b}$)-plane, i. e., the equation $\mathbf{n}\boldsymbol{\nu}(t_0) = 0$ being satisfied

$$\mathbf{n} = \cos\chi\, \boldsymbol{\tau} + \sin\chi\, \mathbf{b}, \quad \mathbf{n}\boldsymbol{\nu}(t_0) = 0, \tag{8}$$

where χ is the angle between the vectors $\boldsymbol{\tau}$ and $\mathbf{n}$.

The polarization unit vectors $\mathbf{e}_\pi$, $\mathbf{e}_\sigma$ on the plane that is perpendicular to line of sight are

$$\mathbf{e}_\sigma = \boldsymbol{\nu}, \quad \mathbf{e}_\pi = \sin\chi\,\boldsymbol{\tau} - \cos\chi\,\mathbf{b}, \quad [\mathbf{e}_\sigma, \mathbf{n}]. \tag{9}$$

Expanding the position vector $\mathbf{r}(t)$ into a Taylor series about $(t - t_0)$, and then substituting in (7), we obtain [8]

$$E_i(\omega) = \frac{-i\omega e_\alpha}{cR_0}\exp\{i\Phi\}\int_{-\infty}^{\infty}\left\{\begin{array}{c}\beta\sin\chi \\ -kv^2(t-t_0)/c\end{array}\right\}\times$$
$$\times\exp\{i\omega[(1-\beta\cos\chi)(t-t_0)+$$
$$+\left(k^2v^3\cos\chi - \varkappa k v^3\sin\chi\right)(t-t_0)^3/6c]\}dt, \tag{10}$$

where $i = \pi, \sigma$, the top string is related to π- polarization, and $\Phi = \omega[R_0/c + t_0 - \mathbf{nr}(t_0)/c]$ is the constant phase.

As shown in [8], it is possible to neglect the term $\varkappa k v^3 \sin\chi$, and we have

$$\frac{dE_\pi}{do\,d\omega} = \frac{e_\alpha^2\omega^2}{3\pi^2 c}\frac{\beta^2}{k^2v^2\gamma^4}\psi^2\left(1+\psi^2\right)K_{1/3}^2(\eta), \tag{11}$$

$$\frac{dE_\sigma}{do\,d\omega} = \frac{e_\alpha^2\omega^2}{3\pi^2 c}\frac{\beta^2}{k^2v^2\gamma^4}\left(1+\psi^2\right)^2 K_{2/3}^2(\eta), \tag{12}$$

where $\psi = \gamma\chi$, $\eta = (1/2)(\omega/\omega_c)(1+\psi^2)^{3/2}$, $\omega_c = (3/2)\gamma^3 kv$, $K_{1/3}(x)$, $K_{2/3}(x)$ are the modified Bessel functions.

Since the instantaneous curvature radius changes as the particle moves from one to another trajectory points, the synchrotron radiation mechanism in circular magnetic field differs from synchrotron radiation in straight magnetic lines. Let us consider the radiation pattern.

As it is known, the radiation of an ultrarelativistic charge is concentrated into a cone along the particle velocity. When the charge drives along trajectory (2), the instantaneous direction of the radiation beam changes. As a result, the radiation will be concentrated in the neighborhood of a surface (design it by *S*), which generating lines coincide with the velocity vectors. The intersection of the surface *S* with the unit sphere gives a line *L*. Points at line *L* correspond to directions at different instants of time t_0.

In the plane, which is perpendicular to the line *L*, the form of radiation pattern is described by equations (11), (12) with the curvature radius at time t_0 being taken as the circle radius. For σ - polarization the radiation has maximum in directions at the line *L*, $\chi = 0$; the radiation in π - polarization has peaks for angles $|\chi| = 1/\gamma$ and tends to zero at line *L*, ($\chi = 0$).

Let us consider the radiation pattern, assuming that the direction of emission passes near to *x*-axis. Introduce angular coordinates θ_y and θ_z, where θ_y is the angle between $\mathbf{n}$ and (*x*, *z*)-plane, and θ_z is the angle between *x*-axis and the projection of the vector $\mathbf{n}$ onto (*x*, *z*)-plane.

The projection of line *L* on (*y*, *z*)-plane in the case of small angles $\theta_y << 1$, $\theta_z << 1$ is given by equations

$$\theta_y \approx v_y/v = -\frac{\Omega}{\omega_B}\left(\omega_B t_0 + q\sin\omega_B t_0\right)$$
$$\theta_z \approx v_z/v = -\frac{\Omega}{\omega_B}\left(1 + q\cos\omega_B t_0\right). \tag{13}$$

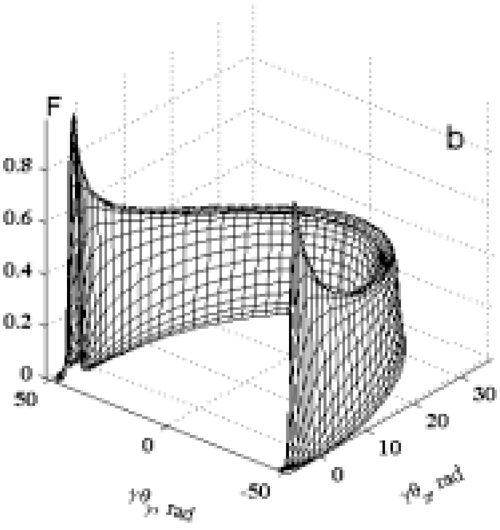
The form of the radiation pattern depends on the relation between the velocity of centrifugal drift $|v_D|$ and Larmor velocity, $|v_L| = |\omega_B| r_B$, $q \equiv |v_L/v_D|$.

In case $q >> 1$ the radiation pattern resembles the radiation cone (with the apex angle $\sim 1/\gamma_\parallel$ and angular width $\sim 1/\gamma$ for the cone wall) in straight magnetic field. In the case $q << 1$ we have the limit of curvature radiation. In both cases $q >> 1$ and $q << 1$, the curvature radius does not depend on time t_0 so that the profiles of radiation pattern remain constant.

In case $q \sim 1$ the spectral angular distribution of radiation has specific features as compared with the synchrotron radiation mechanism in homogeneous magnetic field. For $q \sim 1$ the drift velocity is approximately equal to Larmor velocity $|v_D| \cong |\omega_B| r_B \cong c/(\sqrt{2}\gamma_{\|})$ so that the curvature radius varies with time t_0. There appears the range of characteristic frequencies from $\omega \sim \gamma^3 \Omega |1-q|$ up to $\omega \sim \gamma^3 \Omega (1+q)$ instead of one characteristic frequency ($\omega \sim \gamma^3 q \Omega$ for an emission in straight magnetic field lines, or $\omega \sim \gamma^3 \Omega$ for a curvature radiation).

In Fig.1 the radiation patterns (for σ-+π-polarization) at frequencies corresponding to minimal (Fig.1a) and maximum (Fig.1,b) curvature radius of trajectory (2) at $q = 1{,}2$, $\gamma\delta = 15$, $-\pi \le \omega_B t_0 \le \pi$ are represented. The picture is periodically repeated with time along θ_y-axis, Fig.2,a. If the frequency of radiation corresponds to the maximum characteristic frequency $\omega/\omega_c = 0{,}8338(1+q)$, the radiation pattern has peaks for directions corresponding to trajectory points with minimal curvature radius, $\theta_y = 2\pi\delta n$, $n = 0, \pm 1, \ldots$; $\theta_z = -(1+q)\delta$. Denote these directions by *A*. At higher frequencies, the radiation is more concentrated in the neighborhood of directions *A*.

At the minimal characteristic frequency $\omega/\omega_c = 0{,}8338|1-q|$, Fig. 1,b, the peaks of the radiation pattern correspond to the trajectory points with maximal curvature radius, $\theta_y = -\pi\delta + 2\pi\delta n$, $n = 0, \pm 1, \ldots$; $\theta_z = (-1+q)\delta$, Fig. 2,b. For lower frequencies the radiation is even more concentrated in the neighborhoods of these points. When the direction of light biases to *A*, the section of radiation pattern becomes two-humped because of increasing the relative contribution of π-polarization component in the total (π- + σ-) radiation beam, Fig. 1,b.



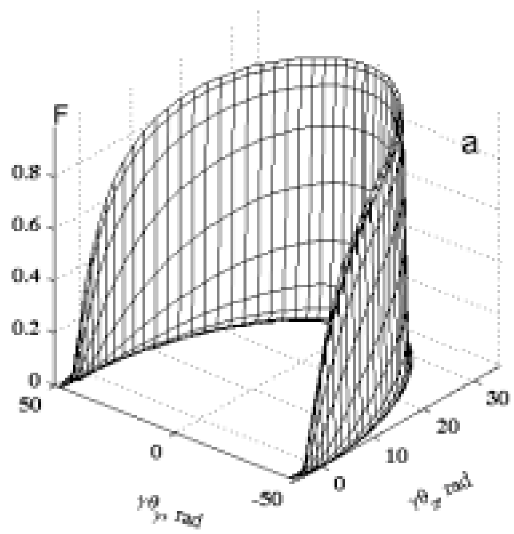


Fig. 1 Radiation pattern at the given frequency ω :
a) ω is equal to the maximal characteristic frequency;
b) ω equals minimal characteristic frequency;

$$F \equiv \frac{d(E_\sigma + E_\pi)}{do d\omega} \Bigg/ \left(\frac{dE_\sigma}{do d\omega}\right)_{\max}$$

Thus the form of radiation pattern depends on the frequency of radiation while the particle moves along trajectory (2).

Polarization properties of radiation are described by equations (9), (10). The ort $\mathbf{e}_\sigma$ coincides with the tangent to line *L*, Fig. 2, $\mathbf{e}_\pi$ is perpendicular to $\mathbf{e}_\sigma$ and **n.** For the directions at line *L* the radiation has linear polarization ($E_\sigma \ne 0, E_\pi = 0$). The sense of elliptical polarization coincides with the sense of particle rotation around **n**.

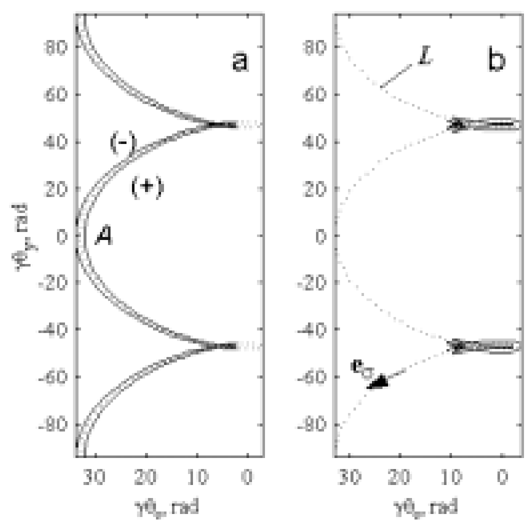


Fig.2. The width of radiation pattern. (solid line) is the contour at level of one half of maximal value; (+),(-) denote the sense of elliptical polarization

In the case of exact equality $q=1$ the trajectory (2) has points at which the curvature becomes equal to zero. At these point equation (10) is failed. In [8] it was found that the approximation (10) is correct, if $|q-1|>(|\omega_B|/(\gamma\Omega))^{1/2} \; =(\gamma_{\|}/\gamma)^{1/2}$. The case $q=1$ needs a special study. This will be the object of another paper.

4. RADIATION SPECTRUM

To find the radiation power per unit frequency, we shall integrate equations (11), (12) over the radiation angle and then divide it by the time interval of radiation. Let μ and χ be angular coordinates. The variable μ describes directions that correspond to segments of line *L*. The angle χ corresponds to arcs of the great circle, which is perpendicular to line *L*. $d\mu = dv/v = |\ddot{\mathbf{r}}|\, dt_0/v = kv\, dt_0$, and the element of solid angle has the form

$$do = d\chi d\mu = d\chi kv\, dt_0 . \tag{14}$$

Dividing (11), (12) by $2\pi/|\omega_B|$ and integrating over a solid angle, we obtain

$$\frac{dP}{d\omega} = \frac{3\sqrt{3}}{4\pi}\frac{\beta^2}{\gamma^3}\int_0^{\pi}\frac{d(|\omega_{\mathrm{B}}|t)}{\pi}\frac{W(k)}{kc}y\int_y^{\infty}dxK_{5/3}(x), \tag{15}$$

where $y=\omega/\omega_c$, $W(k)=(2/3)(e_\alpha^2/c)\gamma^4k^2v^2$. The expression after the first integral sign in (15) can be interpreted as a spectral power of radiation for a charged particle moving in a circular orbit with the instantaneous radius $r_{\mathrm{C}}=k^{-1}(t)$.

Let us derive formula (15) without using expression (14). At first we integrate over solid angle in (6).

4.1. SCHWINGER'S FORMULA

Substituting (7) in equation (6), we obtain

$$\frac{dE}{d\omega do} = \int_{-\infty}^{\infty}dt_1dt_2\frac{e_\alpha^2\omega^2}{4\pi^2c}[\boldsymbol{\beta}_1\boldsymbol{\beta}_2-(\mathbf{n}\boldsymbol{\beta}_1)(\mathbf{n}\boldsymbol{\beta}_2)]\times \\ \times\exp\{i\omega[t_2-t_1-\mathbf{n}(\mathbf{r}_2-\mathbf{r}_1)]\}, \tag{16}$$

where $\mathbf{r}_i=\mathbf{r}(t_i)$, $\boldsymbol{\beta}_i=\boldsymbol{\beta}_i(t_i)$, $i=1,2$.

Integrating by parts the second term in (16), then integrating in do, and introducing the variable $\tau=t_2-t_1$, we obtain (see also [12])

$$\frac{dE}{d\omega} = -\frac{e_\alpha^2\omega}{\pi}\int_{-\infty}^{\infty}dtd\tau\left[1-\frac{\mathbf{v}(t+\tau)\mathbf{v}(t)}{c^2}\right]\cos\omega\tau\times \\ \times\frac{\sin\omega|\mathbf{r}(t+\tau)-\mathbf{r}(t)|/c}{|\mathbf{r}(t+\tau)-\mathbf{r}(t)|}. \tag{17}$$

From expression (17) follows the expression for a spectral power at the time t

$$\frac{dP(t)}{d\omega} = -\frac{e_\alpha^2\omega}{\pi}\int_{-\infty}^{\infty}d\tau\left[1-\frac{\mathbf{v}(t+\tau)\mathbf{v}(t)}{c^2}\right]\cos\omega\tau\times \\ \times\frac{\sin\omega|\mathbf{r}(t+\tau)-\mathbf{r}(t)|/c}{|\mathbf{r}(t+\tau)-\mathbf{r}(t)|}. \tag{18}$$

It is the formula (I.37) obtained by Schwinger in [3]. He considered the rate at which the electron does work on the radiation field. Equation (18) is the starter formula in [6]. Let us now show that (15) is also followed from equation. (18). Using the Frenet formulae, we obtain

$$|\mathbf{r}(t+\tau)-\mathbf{r}(t)| = \tau\left(1-\frac{\tau^2}{24}k^2v^2\right), \tag{19}$$

$$1-\frac{\mathbf{v}(t+\tau)\mathbf{v}(t)}{c^2} \cong 1/\gamma^2+\frac{\tau^2}{2}k^2v^2 . \tag{20}$$

Substituting equations (19), (20) into (18), we reduce expression (18) to

$$\frac{dP(t)}{d\omega} = -\frac{e_\alpha^2\omega}{\pi\gamma^2v}\left\{\int_0^{\infty}\frac{d\tau}{\tau}\left(1+\frac{\tau^2}{2}\gamma^2k^2v^2\right)\times\right. \\ \left.\times\sin\frac{\omega\tau}{2\gamma^2}\left(1+\frac{\tau^2}{12}\gamma^2k^2v^2\right)-\int_0^{\infty}\frac{d\tau}{\tau}\left(1+\frac{\tau^2}{2}\gamma^2k^2v^2\right)\sin\omega\tau\right\} \tag{21}$$

where $k=k(t)$.

After introducing in (21) the new integration variable $x=\tau\gamma kv$ and employing the formula from [3],

$$\int_0^{\infty}(1+2x^2)\sin\frac{3}{2}y(x+\frac{x^3}{3})\frac{dx}{x}-\frac{\pi}{2}=\frac{1}{\sqrt{3}}\int_y^{\infty}dxK_{5/3}(x),$$

we obtain expression (15).

4.2. GENERALIZATION OF RADIATION SPECTRUM

To integrate with respect to $|\omega_B|t_0$ in (15), we introduce the variable $z=1+q^2+2q\cos\omega_Bt_0$ and change the order of integration. Then [8]

$$\frac{dP}{d\omega} = \frac{P_{\mathrm{C}}}{\omega_{\mathrm{C}}}f(y_{\mathrm{C}},q), \tag{22}$$

$$f(y_{\mathrm{C}},q) = \frac{9\sqrt{3}}{8\pi}y_{\mathrm{C}}\left\{\int_{\frac{y_{\mathrm{C}}}{|1-q|}}^{\infty}dxK_{5/3}(x)+\right. \\ \left.+\frac{1}{\pi}\int_{\frac{y_{\mathrm{C}}}{1+q}}^{\frac{y_{\mathrm{C}}}{|1-q|}}dxK_{5/3}(x)\left(\frac{\pi}{2}+\arcsin\frac{1+q^2-y_{\mathrm{C}}^2/x^2}{2q}\right)\right\},$$

where $P_{\mathrm{C}}=\dfrac{2}{3}\dfrac{e_\alpha^2}{c}\gamma^4\beta_{\|}^2\Omega^2$ is the total power emitted by a charged particle moving with velocity $v_{\|}$ along a circular orbit of radius *R*, $y_{\mathrm{C}}=\omega/\omega_C$.

Thus, the universal function of synchrotron radiation for a relativistic electron moving in circular orbit [2, 3, 12]

$$f(y) = \frac{9\sqrt{3}}{8\pi}y\int_y^{\infty}dxK_{5/3}(x) \tag{23}$$

is replaced by expression (22) for a relativistic electron moving along the spiral trajectory in circular magnetic field. In [6], the radiation spectrum, which form is different from (22), was obtained from the Schwinger formula (18). As it has shown above, spectrum (22) also follows from (18). The radiation spectrum obtained in [6] is given in Appendix. Thus (22) and the corresponding formula in [6] are two different representations of the radiation spectrum.

Integrating in (22) with respect to frequency, we obtain the total emitted power

$$P = \frac{2}{3}\frac{e_\alpha^2}{c}\gamma^4\beta_\parallel^2\Omega^2\left(1+q^2\right). \qquad (24)$$

The same form has the expression for power losses of a relativistic electron moving along the circular trajectory of effective radius $R/\sqrt{1+q^2}$.

Equation (22) at $q << 1$ и $q >> 1$ reduces to formulae of the curvature radiation and synchrotron radiation for spiral trajectory in straight magnetic field, respectively. In these cases, the first integral in (22) is smaller than the second one. The most essential difference from the case of synchrotron radiation in straight magnetic field arises, if $q \sim 1$, and then the second integral in (22) is larger than the first.

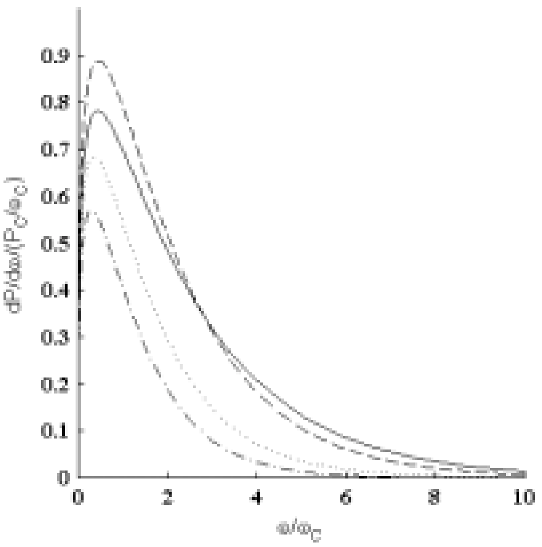


Fig.3. Universal functions of synchrotron radiation: (solid line) is the spectrum (23); (dotted line) the spectrum in straight magnetic field lines; (dot-and-dash line) the curvature radiation spectrum; (dashed line) synchrotron radiation for an electron having the circular trajectory with effective radius $R/\sqrt{1+q^2}$

Let us compare exact expressions for spectrum (22) and total energy losses (24) with approximate expressions (usually used at interpretation of experimental data), in which formula (23) is used. In Fig. 3 we compare different radiation mechanisms when $q \sim 1$. a) The curvature of magnetic force lines is not taken into account, and the formula for synchrotron radiation of an electron moving with the pitch angle $\sin\psi_P = \omega_B r_B / c$ in straight magnetic field is used. In this case $y_c = \omega/[(3/2)\gamma^3 \,|\, \omega_B \,|\, \sin\psi_P]$ and the spectrum is described by the first integral with the lower limit of integration y_c / q in (22), (dotted line in Fig.3). The total emitted power is proportional to q^2. b) We neglect pitch angles and consider the curvature radiation for an electron moving along the circular magnetic line with curvature radius *R*. This spectrum is described by the first integral, which has the lower limit of integration y_c, in (22). The spectrum of curvature radiation is plotted by the dot-and-dash curve. The total power losses is described by the first term in (24). c) As it was already mentioned above, the total power loss for particles in curved magnetic field (24) coincides with the power loss of a relativistic electron having circular trajectory of radius $R/\sqrt{1+q^2}$ (dashed line in Fig.2).

Considering the graphs such as represented in Fig.3 at various values of parameter *q*, we find that the differences between spectrum (22) and the spectrum of synchrotron radiation in straight magnetic field are essential if *q* belongs to the interval, $0{,}2 < q < 5$. Thus when $q \sim 1$, it is necessary to use formula (22). The derived formulae, strictly speaking, are obtained when condition [8] $|q-1| > (|\omega_B|/(\gamma\Omega))^{1/2} = (\gamma_\parallel/\gamma)^{1/2}$ is taken place. These conditions can be fulfilled both in astrophysical, and in laboratory plasma.

APPENDIX

Averaging (21) over time $T = 2\pi/|\omega_B|$ and by taking expansion

$$e^{iz\sin t} = \sum_{-\infty}^{\infty} e^{\mathrm{int}} J_n(z)\,,$$

where $J_n(z)$ are Bessel functions, we obtain

$$\frac{dP}{d\omega} = \int_0^{2\pi} \frac{d\,|\,\omega_B\,|\,t}{2\pi}\frac{dP(t)}{d\omega} =$$

$$= -\frac{e_\alpha^2\omega}{\pi\gamma^2 v}\left\{\int_0^\infty \frac{d\tau}{\tau}\left(1+\frac{\gamma^2\Omega^2\tau^2}{2}(1+q^2)\right)\times\right.$$

$$\times J_0\left(\frac{\omega\Omega^2\tau^3}{12}q\right)\sin\frac{\omega\tau}{2\gamma^2}\left(1+\frac{\gamma^2\Omega^2\tau^2}{12}\left(1+q^2\right)\right)-$$

$$-\int_0^\infty \frac{d\tau}{\tau}\gamma^2\Omega^2\tau^2 q J_0'\,(yqx^3)\times$$

$$\left.\times\cos\frac{\omega\tau}{2\gamma^2}\left(1+\frac{\gamma^2\Omega^2\tau^2}{12}(1+q^2)\right)-\frac{\pi}{2}\right\}.$$

Replacing the variable of integration $x = \gamma\Omega\tau/2$ and introducing $y = \omega/((3/2)\gamma^3\Omega)$, we obtain expression (14) from the paper [6].

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