HIGH NUMBER HARMONIC EXITATION BY OSCILLATORS IN PERIODIC MEDIA AND IN PERIODIC POTENTIAL

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The some results of theoretic and experimental investigations about excitation high number harmonics by non-relativistic oscillators are represented in this report. Were shown that in media, which has even small periodic heterogeneity of dielectric permeability or potential, non-relativistic oscillators can radiate as relativistic particles. They can efficiency radiate high number harmonics. The theory as one particle radiation as selfconsistent nonlinear theory radiation of oscillator ensemble was created. The experimental results confirm the main results of theory. In particularly, there was exited ultraviolet radiation in experiment, when on a crystal intense ten-centimetric radiation was acting.

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1. INTRODUCTION

The expression for radiation capacities of charge particles, which are moving in media with dielectric permeability $\varepsilon = \varepsilon_0 + q \cdot \cos(\vec{\kappa} \cdot \vec{r})$ along the trajectory $\vec{r} = \vec{V}_0 t + \vec{r}_0 \sin(\Omega \cdot t)$ was found in previous investigations [2-4]. It necessary to mark, that the radiation of relativistic particles in periodic inhomogeneous were studied by many authors (see, for example [5-9] and quoted literatures there). This radiation is interesting because using it open new opportunity to excite short wave radiation $\lambda \sim d / \gamma^2$ with high efficiency. Such opportunity arises due Doppler effect. Such radiation is already used in counters of the charged particles. Moreover, it is supposed that this radiation can be useful as sources of intense x-ray radiation [9].

2. RADIATION OF ONE CHARGED PARTICLE

Below we shall be interested only radiation of the non-relativistic particles $\beta << 1$. The radiation features of such particles, as it seems to us, are bigger interest for using this radiation as sources of the intense short wave radiation. It is necessary to say, that non-relativistic particles radiate “long wave” radiation. The wavelength λ of such radiation is large than the period heterogeneity of the media d, where the radiation take place $\left(\lambda \sim d / \beta\right)$.

Let’s consider oscillator, which is rest ($V_0 = 0$). Let’s assume, that vectors $\vec{\kappa}$ and $\vec{r}_0$ are parallel to z axis ($\vec{\kappa} || \vec{r}_0 || z$). In this case, such expression for capacities of radiation was received in the work [2]:

$$\frac{\partial W}{\partial t} = \left(\frac{e^2 \Omega^2 \cdot \beta_\perp^2}{3c}\right) \frac{3q^2}{2} \sum_{n=1}^{\infty} \frac{n^4}{m^2} J_n^2(m) \int_0^{\pi} (\sin\theta)^3 d\theta \quad (1)$$

where $\beta_\perp = \dfrac{r_0 \Omega}{c}$, $m = \vec{\kappa} \cdot \vec{r}_0$.

It is useful to compare this expression with expression for capacities of a oscillator radiation in homogeneous media (in vacuum) (see, for example, [1]):

$$\frac{\partial W}{\partial t} = \frac{e^2 \Omega^2}{c} \sum_{n=1}^{\infty} n^2 \int_0^{\pi} J_n^2(n\beta_\perp \cos\theta) \sin\theta \tan^2\theta d\theta . \quad (2)$$

Let's formulate more significant oscillator radiation characteristics in heterogeneous media and compare them to the same characteristics of radiation in homogeneous environment.

Spectrum. In a fig. 1 is submitted the dependence of function $G(n,m) = \left[\frac{n^2}{m} J_n(m)\right]$ from harmonic numbers n at $m = 1000$ $\beta = 0.1$ $(\gamma = 1.005)$, $q = 10^{-5}$. The function $G(n,m)$ determines dependence of radiation capacity of oscillator from number of the harmonic. It is visible, that the radiation capacity grows with growth of harmonic number. The radiation maximum is at the harmonic number $n \sim m = 1000$. It is significant, that, as it follow from the equation (2), it is necessary to have energy of oscillator $\gamma > 500$ in vacuum for receiving such radiation intensity on this harmonic. Thus, the small dielectric permeability perturbation of media ($q = 10^{-5}$), where radiation take place, can result in qualitative change of the radiation spectrum of the such oscillator. The radiation spectrum of such oscillator becomes similar to a radiation spectrum of relativistic oscillator in homogeneous media.

The directivity diagram. As it follow from equation (1), the directivity radiation diagrams of non-relativistic oscillator for all harmonics are coincide with directivity diagram of a dipole. It necessary to mark, that a directivity diagram of a relativistic oscillator is nestles on a oscillator trajectory for high number harmonic.

Polarization. The analyze radiation field structure of a non-relativistic oscillator in periodic inhomogeneous media show, that polarization of this radiation is not differ from the radiation polarization of a oscillator in homogeneous media. It is useful take in mind, that energy of the oscillator must be more than the energy of radiated quantum ($E = mv^2 / 2 > \hbar\omega$), when analyze of radiation take place.

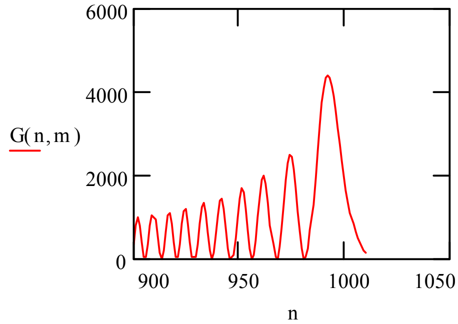

Fig.1. Efficiency of oscillator radiation on high number harmonics

Radiation in periodic potential. Below we will be interested with radiation of oscillators in crystals. In a crystal there is a periodicity not only in dielectric permeability, but also periodicity of a crystal lattice potential. One more mechanism of radiation in this case is possible which is very similar on described above and in works [2-4]. We shall briefly describe this mechanism. Let charged particle goes in external periodic in time an electric field $E(t) = E \cdot \cos(\Omega \cdot t)$ and in a field of periodic in space potential. $U = U_0 + g \cdot \cos(\kappa \cdot z)$.

For simplicity we shall consider, that the movement occurs only along an z axis. Let's consider, that of intensity of these fields are small enough, so it is possible to consider movement of a particle in these fields as non-relativistic. Besides we shall consider that $E >> g \cdot \kappa$. In this case it is possible to present function, which describes displacement of a particle along an z axis, as the following line:

$$z(t) = -\frac{A \cdot c}{\Omega}\cos(\Omega t) - \sum_{j=0}^{\infty}\frac{B}{(2j+1)^2 \cdot \Omega} \cdot J_{2j+1}\left(\frac{\kappa c}{\Omega}A\right) \cdot \sin\left[(2j+1)\cdot\Omega\cdot t\right]\cdot(-1)^{2j+1} \qquad (3)$$

where J_{2j+1} - Bessel function - $(2j+1)$ - order , $A \equiv (eE)/(mc\Omega) = \beta$, $B \equiv (eg\kappa)/mc\Omega$.

Using the formula (3), it is easy to find radiation intensity of the charged particle. We are interested with radiation of high number harmonics. For high numbers ($j >> 1$) the amplitudes of the Furies component of the line (3) quickly decrease with growth number.

The exception is addend, in which the argument of Bessel function is equal to number of Bessel function: $\kappa c A = (2j+1)\Omega$. Taking into account only this component, we shall receive the formula (2) for capacity of radiation, in which it is necessary to put $r_0 = B \cdot c \cdot \Omega \cdot J_m(m)/\omega^2$, $m = (2j+1)$, $\omega = m \cdot \Omega$. Being limited in the formula (2) by dipole approximation, we shall receive the following expression for radiation capacity:

$$\frac{\partial W}{\partial t} = \frac{e^2\Omega^2}{3 \cdot c}B^2 \cdot J_m^2(m)$$

$$= \frac{e^2\Omega^2}{3 \cdot c}\left(\frac{eg}{mc^2}\right)^2\frac{1}{A^2} \cdot m^2 J_m^2(m) \qquad (4)$$

Thus, we see, that conditions for maximum radiation in this case completely coincides with a condition of oscillator radiation in periodically non-uniform dielectric, i.e. both in that and in the other case the maximum of radiation corresponds to the same frequency. When $(eg/mc^2) > (qA^2)$ the role of periodic potential on radiation will be more significant.

Quantum consideration. The significant information about radiation features of the charged particles in periodic potential can be received at use quantum electrodynamics methods. Using a perturbation method, it is easy to receive the following expression for radiation capacity of the charged particle:

$$P = \int_0^{2\pi} d\varphi \int_0^{\pi} d\theta \int d(\hbar\omega) \cdot W_{f,i}\,\rho(\hbar\omega)\sin\theta$$

here

$$W_{f,i} = \frac{2\pi}{\hbar}|U_{f,i}|^2\,\delta(\hbar\omega - \Delta E),$$

$$\rho = \frac{L^3\omega^2 n^3}{(2\pi c)^3\hbar},$$

$$H_{f,i} = \frac{dh}{m}\sqrt{\frac{2\pi h}{L^3 w_\lambda}} \cdot \int\sum|\Psi_{N+1}|^2 \cdot \sqrt{N+1} \cdot$$

$$\left[(\vec{e}_\lambda\vec{k})(\Psi_f^*\Psi_i) + \vec{e}_\lambda(\Psi_f^*\hat{\vec{p}}\Psi_i)\right]\exp\left(-i\Delta\vec{k}\cdot\vec{r}\right)d\vec{r} \quad , \qquad (5)$$

$n^2 = \varepsilon$ -dielectric permeability of media, $n-$ its parameter of refraction, $\Delta\vec{k} = \vec{k}_i - \vec{k}_f - \vec{k}_\lambda$

If particle goes in potential with weak periodic heterogeneity, its wave function is possible to present as:

$$\Psi_i = \sum_m \Psi_{i,m}\exp\left[i\left(\vec{k}_i + m\cdot\vec{\kappa}\right)\cdot\vec{r}\right]. \qquad (6)$$

where $\Psi_{i,m} \sim g^m \cdot \Psi_{i,0}$

From (6) it is visible, that the wave function has addends, which can be identified with particles, which speed are large than speed of a real particle. Such addends can be identified with fast virtual particles. In themselves they do not exist. Analogy to virtual waves in periodically non-uniform media is looked through. Only in the latter case we were interested with slow virtual waves. For a case of particles we will be interested with fast virtual particles. Substituting wave function (6) in the formula (5), it is possible to receive the following expression for radiation capacity of the charged particle, which goes in periodic potential:

$$P = g^2 \cdot \frac{q^2 V}{c^2}(N+1)\int\omega \cdot d\omega\left(1 - \frac{c^2}{v^2\varepsilon}\right) \cdot \qquad (7)$$

where $v = 2\cdot\omega/(\vec{\kappa}\cdot\vec{e}_i)$ when $v_i >> v_f$; $v = \omega/(\vec{\kappa}\cdot\vec{e}_i)$ when $v_i \sim v_f$; e_i - unit vector directed along vector $\vec{v}_i$.

If in periodic potential goes oscillator, we receive the formula, which coincides with the formula (4).

3. RADIATION OF THE OSCILLATOR FLOW

At a research of the elementary mechanism of the radiation of the charged oscillator, which moves in a periodically inhomogeneous medium the possibility of the radiation high number harmonics by nonrelativistic oscillator was shown. In order the such a radiation to be effective the following condition must be satisfied: $d \approx \beta\lambda,\ r_o \approx nd/2\pi$, here λ is the wave length of the radiation, d - period of a heterogeneity, r_0 is the amplitude of a oscillator displacement from a position of an equilibrium, $\beta = v/c << 1$, v - velocity of the oscillator, n is the number of radiated harmonic. In present section we study self-consistence process of the excitation of electromagnetic radiation by an ensemble of charged oscillators both analytical and numerical methods. The dispersion equation and the increments for excited waves are obtained. The analytical results are confirmed by computer simulation.

Basic equations. We consider excitation of an electromagnetic wave by an ensemble of oscillators in a periodically inhomogeneous medium which is described by a dielectric permeability:

$$\varepsilon = 1 + 2q\cos\kappa z, q << 1. \quad (8)$$

Most completely process of interaction of the charged particles with excited fields is described by self-consistent model, which consists of Maxwell equations for fields and motion equations for particles in these fields.

$$\frac{\partial\vec{B}}{\partial t} = -c\,\mathrm{rot}\,\vec{E}, \quad \frac{\partial\vec{D}}{\partial t} = c\,\mathrm{rot}\,\vec{H} - 4\pi\vec{j}$$
$$\frac{d\vec{p}}{dt} = e\vec{E} + \frac{e}{c}\vec{v}\times\vec{B} + \vec{F}_0\sin\Omega t, \quad \frac{d\vec{r}}{dt} = \vec{v} \quad (9)$$

where $\vec{D} = \varepsilon\cdot\vec{E}$, $\Omega -$ oscillation frequency of oscillators, F_0 - amplitude of a force which acts on oscillators. Oscillations of oscillators happen along axes Z.

At a research of the elementary mechanism of a radiation of an oscillator in a periodically inhomogeneous medium was shown that the radiation of an oscillator is mainly directed across its motion. Therefore we shall search for a solution for the raised wave as

$$\vec{E} = \mathrm{Re}\,\vec{A}(t,z)\exp(ikx). \quad (10)$$

It is known, that in periodic inhomogeneous mediums it is possible to search for a solution as expansion on spatial harmonics of a heterogeneity, therefore we can write (10) as

$$\vec{E} = \mathrm{Re}\sum_l \vec{E}_l(t)\exp(ikx + il\chi z). \quad (11)$$

Let's study temporal evolution of an electromagnetic field (4) with distinct from of zero by components E_x, E_z, H_y. Substituting expressions for fields (4) in the system (2) and averaging over spatial phase of perturbation we receive the following set of equations for fields and oscillators:

$$\frac{dp_x}{d\tau} = \mathrm{Re}\sum_l \varepsilon_{x,l}\exp(ikx + ilxz) -$$
$$- v_z\,\mathrm{Re}\sum_l h_{y,l}ex(ikx + ilxz),$$

$$\frac{dp_z}{d\tau} = \mathrm{Re}\sum_l \varepsilon_{z,l}\exp(ikx + ilxz) +$$
$$+ v_x\,\mathrm{Re}\sum_l h_{y,l}\exp(ix + il\kappa z) + f_0\cos\Omega\tau$$

$$\frac{dx}{d\tau} = v_x, \quad \frac{dz}{d\tau} = x_z, \quad \vec{v} = \vec{p}/\sqrt{1 + p_x^2 + p_y^2},$$
$$\frac{dh_{y,l}}{d\tau} i\varepsilon_{z,l} - ilx\varepsilon_{x,l}, \quad (12)$$
$$\frac{d}{d\tau}(\varepsilon_{x,l} + q\varepsilon_{x,l-1} + q\varepsilon_{x,l+1}) =$$
$$= -ilxh_{y,l} - \frac{2w_b^2}{(2\pi)^2}\int_0^{2\pi}\int_0^{2\pi} v_x\exp(-ix - ilxz)dx_o dxz_o$$

$$\frac{d}{d\tau}(\varepsilon_{z,l} + q\varepsilon_{z,l-1} + q\varepsilon_{z,l+1}) =$$
$$= -ih_{y,l} - \frac{2w_b^2}{(2\pi)^2}\int_0^{2\pi}\int_0^{2\pi} v_x\exp(-ix - ilxz)dx_o dxz_o$$

.

The integration on the right-hand sides of equations for fields is over the initial values of the oscillator coordinates. The set of equations (12) are written in dimensionless variables:

$$kct \to \tau,\ k\vec{r} \to \vec{r},\ \frac{\vec{p}}{mc} \to \vec{p},$$

$$\kappa/k \to \kappa,\ \Omega/kc \to \Omega,\ \vec{\varepsilon} = \frac{e\vec{E}}{mckc},$$

$$\vec{h} = \frac{e\vec{H}}{mckc},\ f_0 = \frac{F_0}{mckc},\ \omega_b^2 = \frac{4\pi e^2 n_b}{m(kc)^2}$$

where m, e - mass and charge of an electron, n_b – density of oscillators.

The analysis of linearized set of equations. The dispersion equation. Let's research the set of equations (12) on a stability in linear approximation on fields. For this purpose we shall present a dependence of fields from time proportional to $\exp(-i\omega t)$ and shall neglect by terms of second order of perturbation. Also we shall consider non relativistic oscillators and we shall leave only main wave E_0 (own wave of the system) and the wave E_1 which corresponds to the first order of a diffraction, i.e. we shall choose the field in following form

$$\vec{E} = \mathrm{Re}\,\vec{E}_0 \exp(ikx - i\omega t) +$$
$$+ \mathrm{Re}\,\vec{E}_1 \exp(ikx + i\kappa z - i\omega t)$$

Fulfilling necessary transformations we receive the system of linear algebraic equation for amplitudes of fields. To have non-zero solution the determinant of this system must be equal to zero. It is dispersion equation. It is tremendous large. That is why we shall represent this equation in most interesting case when the conditions $\kappa >> k$, $\beta << 1$, $\omega_b << \omega \approx kc$ are satisfied. It is natural, that the maximum increment of instability is reached in the case when frequencies of excited waves lie near to the resonance frequencies, therefore we can leave in infinite sums only one resonance term $\omega \approx n\Omega$. The magnitude of this term depends on Bessel function of order *n*. At large numbers *n* Bessel function fast decreases and has maximal value only then, when its argument is equal to its number, in this case Bessel function has asymptotic dependence $J_n(n) \approx (n)^{-1/3}$. Therefore we assume that condition $n = \mu$ (i.e. $n\Omega = \kappa\beta c$) is satisfied also. In this case dispersion equation takes form

$$\left(1 - \frac{k^2c^2}{\omega^2} - \frac{\omega_b^2}{\omega^2}\right)\left(1 - \frac{\omega_b^2 J_n^2(\mu)}{(\omega - n\Omega)^2}\right) = q^2 . \quad (13)$$

In Fig.2 the dispersion curves of the considered system, obtained from the equation (13), are presented

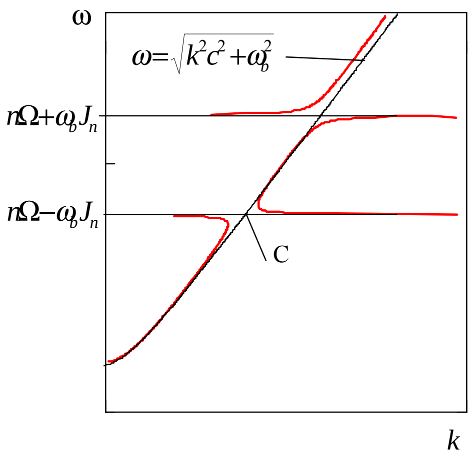

Fig.2.

In the region of intersection of branches of electromagnetic oscillations (point C) the dispersing equation (13) has the complex roots. To determine them we rewrite (13) as

$$(\omega^2 - \omega_0^2)(\omega - \omega_1)(\omega - \omega_2) = q^2\omega^2(\omega - n\Omega)^2 ; \quad (14)$$
$$\omega_0 = \sqrt{k^2c^2 + \omega_b^2},\ \omega_1 = n\Omega - \omega_b J_n,\ \omega_2 = n\Omega + \omega_b J_n$$
.

Assuming that $\omega = \omega_0 + \delta,\ \omega_0 = \omega_1,\ \delta << \omega_b J_n$ we obtain from (11) the following increment of instability:

$$\mathrm{Im}\delta = \frac{q}{2}\sqrt{\omega_0\omega_b J_n},\ \omega_0 = \sqrt{k^2c^2 + \omega_b^2},\ n\Omega = \omega_0 + \omega_b J_n . \quad (15)$$

Thus the self-consistent set of equations (12) has unstable solution with an increment (15).

The set of equations (12) were studied by numerically. The numerical results are in good agreements with analytical ones.

4. THE SDUDING OF GENERATION HARMONICS MECHANIZM IN EXPERIMENT

A.N Antonov, O.F. Kovpik and E.A. Kornilov executed the experimental researches. First of all, the mechanism of harmonics excitation was investigated in a microwave range. In this series of experiments the plasma electrons, which oscillate in a field of an external electromagnetic, were as oscillator ensemble. The artificial lattice was as periodic inhomogeneous media. The excitation of the third harmonic of a falling wave frequency was studied in experiment. The frequency of this wave was 2.7 Gh. As a whole, the results of the carried out experimental researches are in the good qualitative consent with the theory. The excitation of electromagnetic wave on the third harmonic (8,1 Gh) was observed only at simultaneous presence of plasma and lattice, shipped in it. If the lattice left, the radiation on harmonics was absent. If the plasma left, the radiation also was absent. Moreover, the plasma could be deleted from a lattice on various distances. Thus there is some critical distance (~2mm), since which the signal on harmonics vanishes. The polarization of radiation on character corresponds to dipole radiation. It is in the consent with the theory. The directivity diagram of radiation also is in the consent with the theory: the intensity of radiation in a direction that is perpendicular to lattice considerably surpasses radiation intensity in a direction that is parallel to a surface of a lattice.

Excitation of harmonics under acting of electromagnetic radiation on a crystal. If as periodic structure to use a crystal, it is possible to expect to excite optical, UV and X-ray radiation in the same experimental conditions. For check of such opportunity, the same experimental installation was slightly changed. Namely, the resonator was as a load of a high-frequency path (waveguide). The crystal plates of semi-conductor were located in the resonator. The electronic multiplier fixed the optic radiation from the resonator. The photo - multiplier with the converter (UV into optic) for registration UV. The main result of the carried out experiments consists that in all cases the radiation was observed. The origin of this radiation is possible to explain by the mechanism, investigated by us. As an example on Fig. 3 the characteristic results of experiments are represented. The oscillogram of a high-frequency pulse in the resonator on frequency 2,7 Gh and registered radiation from a crystal ($\lambda \sim 10^{-5}$ sm) in this figure are represented. The strength of electric field was 20 kV/sm. Excitation of radiation on a million harmonics thus was observed.

5. DISCUSSION OF RESULTS AND CONCLUSIONS

The main interest represents the results of experimental researches. Therefore, we shall below discuss these results. The results of the carried out experiments, as a whole, are in the good agreement with our representations about the mechanism of high numbers harmonics radiation by non-relativistic oscillators. In many cases there is a good enough quantitative consent of the theory with experiment. It is necessary to note that the results of experiments in a centimetric range are complete enough for unequivocal interpretation. The characteristics of radiation in this range are clear practically in all details. As to a ultra-violet range ($\lambda \sim 10^{-5}\,\mathrm{sm}$) - situation less clear.

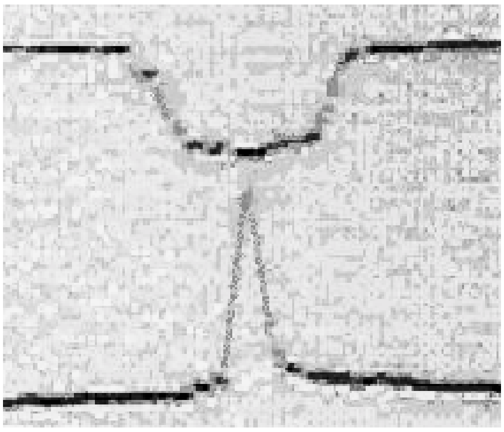

Fig.3. The microwave signal amplitude (the upper ray) and the radiation signal from the crystal (the lower ray)

Unfortunately, we have no sufficient experimental opportunities for more detailed research of this range. Now it is not clear, what role play electrons, taking place near to a surface of a crystal and in its volume.Not clearly also the ratio of the contributions in radiation periodicity of potential and periodicity of dielectric permeability.

Within the framework of the carried out experiments, the displacement formed by an external field oscillation should exceed $10^4 - 10^5$ of atom distances in a crystal. In this case essential role on dynamics of electrons, which are moving in volume of a crystal should be played collisions. The collisions, certainly, will prevent to coherent radiation. If the laser radiation will be use as a wave that forming oscillators, than the necessary displacement of electrons will be only some hundreds atom distances. The role of collisions in this case will be essential smaller. Besides the crystal can be cooled. It is possible to expect, that during formation of radiation, the crystal will not essentially heated.

Now, we don't know other mechanisms (except for researched by us) which could result in radiation, observable by us. Really, such radiation could be caused by discharge. We specially create conditions, in which the discharge is absent. Such radiation could arise as a result of excitation any admixture centers in the semiconductor. However relaxation of the admixture centers carries absolutely other character. It is necessary to notice that when the conditions for existence discharge on a surface of a crystal were created, the intensity of radiation, observable by us, considerably grew. It is possible to explain it both radiation of plasma, and that fact, that the number of the electrons near to a surface of a crystal, in these conditions, was considerably increased. Last fact can result in essential increase of efficiency of the radiation mechanism, considered by us. However, these facts require the further study.

It is necessary to note that the formula (4) for radiation capacity was received in the assumption, that the field of spatially periodic potential is less than a field of an external wave. In many cases it not so. However, as show our preliminary investigation, and in that case, when the return inequality is executed, the spectrum of radiation of the charged particle will have a maximum on the same frequency $\omega \approx \kappa v$. Thus, general feature of the radiation mechanism in periodic media and in periodic potentials is that fact that the particle during quanta radiation can get or give a part of a pulse to periodic structure. It is possible, apparently, to consider that for electrons placed near to a crystal surface fairly to use inequalities, which we used for derivation of the equation (4). For the electrons inside crystal the crystal field considerably exceeds field of an external wave. The special interest represents collective process of radiation. In a centimetric range of lengths of waves we, certainly, observed collective radiation. In optical and ultra-violet ranges we only hope on an opportunity of such radiation. The available experimental results do not give us opportunities to make any conclusion in this occasion.

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REFERENCES

1. A.A. Sokolov, I.M. Ternov. *Relativistic electron* // Moskow. Science. 1974, 392 p.
2. V.A.Buts. Extitation of the harmonics by the oscillators flux in periodically heterogeneous medium //*Intense Microwave Pulses V.* 1997, San Diego, California, v. 31158, p. 202-208.
3. V.A.Buts "Longwave" radiation of charged particles in a media with periodic inhomogeneous // *«Radiotechnic»*. 1997, №9, p. 9-12.
4. V.A.Buts. Shortwave radiation nonrelativistic charged particles // *Journal Tech. Phys.* 1999, v. 69, № 5, p. 132-134.
5. V.L. Ginzburg, V.N.Tsitovich. Transition radiation and transition scattering // Moskow, Science, 1983.
6. V.A.Buts, A.V.Schagin. Investigation of the parametric cherenkov radiation // *Ukraine Phys. Journal.* 1998, v. 43, №9, p.1172-1174.
7. V.G. Barishevskii, I.Ya. Dubovkaya. *Results of a science and engineering. Series: Beams of the charged particles and solid body.* 1992, №4.
8. V.L. Ginzburg // *Progress of Phys. Science*, 1996. v. 166, №10.
9. M.A. Piestrup, P.P. Finman .// *IEEE Journal of Quantum Electronics*. 1983, v. QE-19, № 3.