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WHISTLER WAVE EMISSION BY A MODULATED ELECTRON BEAM ON A METAL-PLASMA BOUNDARY

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The transition radiation of a thin modulated electron beam injected from a conducting plane into a plasma along an arbitrary magnetic field normal to that plane is calculated. The radiation field is formed as a result of the interference of three waves with different wave vectors. The radiation pattern is mainly determined by one of those waves, depending on the parameters of the model.

1. INTRODUCTION

One of the possible ways for interpreting the results of active beam-plasma experiments in the ionosphere is the laboratory simulation of the observed effects. The excitation of waves by modulated electron beams injected in space plasmas belongs to such effects (see, for example, [1]). In the laboratory experiment [2], whistlers excited by a modulated electron beam injected from an electron gun through a metal surface into a magnetoplasma were observed. It was shown that in some cases this excitation occurs via a transition radiation mechanism. The transverse length of the formation zone of the transition radiation was calculated in [3] for conditions typical of the experiment [2]. It has been shown that this length is considerably less than the dimensions of the injector. It means that the model of a radially restricted beam injected from a conductive plane is valid for the calculation of this type of radioemission. This model was studied in the whistler approximation ($\omega \ll \omega_c \ll \omega_p$) in [4]. This report presents the results of the transition radiation calculations obtained with the same model but for arbitrary parameters.

2. MODEL DESCRIPTION

A sharp metal-plasma boundary is treated. The plasma is considered to be cold and the ambient magnetic field is directed along the z-axis. A thin modulated electron beam is injected from the metal plane parallel to the magnetic field, forming the current density wave:

$$\vec{j}(\vec{r},t) = j_m(r)\vec{e}_z \exp[i(\omega t - \chi z)], \qquad \lim_{r\to\infty} j_m(r) = 0,$$

$$j_m(r) = \frac{I_m}{\pi r}\delta(r), \quad \chi_{//} = \omega/v_0 \tag{1}$$

(where ω is the modulation frequency and v_0 is the electron beam velocity). The current density (1) is considered to be given. The permittivity tensor of the cold magnetoactive plasma has the following form:

$$\hat{\varepsilon}(\omega) = \begin{bmatrix} \varepsilon_\perp & -i\alpha & 0 \\ i\alpha & \varepsilon_\perp & 0 \\ 0 & 0 & \varepsilon_{//} \end{bmatrix}, \qquad \varepsilon_\perp = 1 - \frac{\omega_p^{\,2}}{\omega^2 - \omega_c^{\,2}},$$

$$\alpha = \frac{\omega_c}{\omega}\,\frac{\omega_p^{\,2}}{\omega^2 - \omega_c^{\,2}}, \qquad \varepsilon_{//} = 1 - \frac{\omega_p^{\,2}}{\omega^2}, \tag{2}$$

where ω_p is the electron plasma frequency and ω_c is the electron cyclotron frequency.

The problem is solved in two stages. At first the transition radiation of electromagnetic waves by a radially unbounded modulated electron beam forming the current density (3)

$$\vec{j}(\vec{r},t) = \vec{e}_z j_m \exp[i(\omega t - \vec{\chi}\vec{r})],$$

$$\vec{\chi} = \{0, \chi_\perp, \chi_{//}\} \tag{3}$$

is examined. At the second stage, the current (1) is expanded into plane partial waves (3). The contributions of the separated partial waves are added to find out the transition radiation of the modulated electron beam (1).

3. TRANSITION RADIATION OF THE PLANE CURRENT WAVE

It is convenient to use the vector-potential instead of the field components of the emitted electromagnetic wave by imposing the calibration condition φ=0. From the Maxwell's equations one can obtain the wave equation for the vector-potential corresponding to the current density wave (3) taking into account the permittivity tensor (2). Using the vector-potential in the form

$$\vec{A}(\vec{r},t) = \vec{A}'_m \exp(i\omega t - i\vec{\chi}\vec{r})$$

one obtains the relation for the wave as :

$$\vec{\chi}(\vec{\chi}\vec{A}'_m) - \chi^2\vec{A}'_m + k_0^2\hat{\varepsilon}\vec{A}'_m = -\frac{4\pi}{c}\vec{j}_m. \tag{4}$$

Hence the normalized components of the vector –potential amplitude *Am (Ax,Ay,Az)* can be presented as:

$$A_{//} \equiv A_z = -\frac{(n^2-\varepsilon_\perp)(n_{//}^2-\varepsilon_\perp)-\alpha^2}{\Delta},$$

$$A_x = -\frac{n_{//}\left(n_x(\varepsilon_\perp - n^2) + i n_y\alpha\right)}{\Delta},$$

$$A_y = \frac{n_{//}\left(-i n_y(\varepsilon_\perp - n^2) + i n_x\alpha\right)}{\Delta},$$

$$\vec{A}_m = \frac{ck_0^2\vec{A}'_m}{4\pi j_m}, \qquad \frac{\vec{\chi}}{k_0} = \vec{n}, \qquad n_{//} = n_z,$$

$$\Delta = \varepsilon_{//}\left[(n^2-\varepsilon_\perp)(n_{//}^2-\varepsilon_\perp)-\alpha^2\right] + n_\perp^2\left[\varepsilon_\perp(n^2-\varepsilon_\perp)+\alpha^2\right], \tag{5}$$

where Δ=0 is the dispersion relation for eigenmodes of the cold magnetized plasma.

On the metal-plasma boundary the tangential component of the electric field vanishes $\vec{E}_\tau = 0$, and then

$$\vec{A}_\tau = 0,$$

Hence it results in

$$\Sigma_\pm = 0, \tag{6}$$

where

$$\Sigma_+ = \frac{2n_+n_-\left(n^2-\varepsilon_\perp\right)n_{//}A_{//}}{\left(n^2-\varepsilon_\perp\right)\left(n_{//}^2-\varepsilon_\perp\right)-\alpha^2},$$

$$\Sigma_- = \frac{2n_+n_-\alpha n_{//}A_{//}}{\left(n^2-\varepsilon_\perp\right)\left(n_{//}^2-\varepsilon_\perp\right)-\alpha^2}, n_\pm = \frac{n_x \pm in_y}{\sqrt{2}}.$$

Ordinary and extraordinary electromagnetic waves propagating away from the metal plane should also be taken into account besides the current wave. Consequently the boundary conditions on the metal-plasma border can be written as:

$$\begin{cases}\Sigma_{+1}+\Sigma_{+2}+\Sigma_{+B}=0,\\ \Sigma_{-1}+\Sigma_{-2}+\Sigma_{-B}=0\end{cases} \tag{7}$$

where the index *1* refers to the ordinary wave, the index 2 to the extraordinary wave, and the index *B* to the current wave.

The amplitudes of the electromagnetic waves excited by the radially unbounded modulated electron beam have the form:

$$A_{z1,2} = -\frac{\left(n_{//2,1}^2-\frac{1}{\beta^2}\right)\left[\left(n_{1,2}^2-\varepsilon_\perp\right)\left(n_{//1,2}^2-\varepsilon_\perp\right)-\alpha^2\right]}{\left(n_{//1,2}^2-n_{//2,1}^2\right)n_{//1,2}\beta\Delta\left(\frac{1}{\beta}\right)},$$

$$n_{//B} = \frac{1}{\beta}. \tag{8}$$

The transformation coefficient of the current wave into the electromagnetic waves, determined as the ratio of the denominate longitudinal component of the vector-potential and the amplitude of the current density wave, is specified by the formula:

$$K_{1,2}(n_\perp) = -\frac{4\pi}{ck_0^2}\left(n_{//2,1}^2-\frac{1}{\beta^2}\right)\frac{\left(n_{1,2}^2-\varepsilon_\perp\right)\left(n_{//1,2}^2-\varepsilon_\perp\right)-\alpha^2}{\left(n_{//1,2}^2-n_{//2,1}^2\right)n_{//1,2}\beta\Delta\left(\frac{1}{\beta}\right)} \tag{9}$$

4. TRANSITION RADIATION OF THE THIN MODULATED ELECTRON BEAM

After expanding the current (1) into plane partial waves (3) and taking into account (9), one can obtain the expressions for the radiation field components (in cylindrical coordinates):

$$A_{z1,2} = \int_0^\infty \chi_\perp d\chi_\perp \mathrm{K}_{1,2}(\chi_\perp) j_m(\chi_\perp) J_0(\chi_\perp r)\times$$
$$\times\exp\left(i\omega t - i\chi_{//}(\chi_\perp)z\right)$$

$$A_{r1,2} = -\int_0^\infty i\chi_\perp d\chi_\perp K_{1,2}(\chi_\perp) j_m(\chi_\perp) J_1(\chi_\perp r)\times$$
$$\times\frac{\chi_{//}\chi_\perp\left(\chi^2-\varepsilon_\perp k_0^2\right)\exp\left(i\omega t - i\chi_{//}(\chi_\perp)z\right)}{\left(\chi^2-\varepsilon_\perp k_0^2\right)\left(\chi_{//}^2-\varepsilon_\perp k_0^2\right)-\alpha^2k_0^4}$$

$$A_{\varphi1,2} = \int_0^\infty i\chi_\perp d\chi_\perp \mathrm{K}_{1,2}(\chi_\perp) j_m(\chi_\perp) J_1(\chi_\perp r)\times, \tag{10}$$
$$\times\frac{\chi_{//}\chi_\perp\alpha k_0^2\exp\left(i\omega t - i\chi_{//}(\chi_\perp)z\right)}{\left(\chi^2-\varepsilon_\perp k_0^2\right)\left(\chi_{//}^2-\varepsilon_\perp k_0^2\right)-\alpha^2k_0^4}$$

where J_0 and J_1 are the Bessel functions of the zeroth and the first order, respectively. Then the components of that field in spherical coordinates can be written as:

$$A_{R1,2} = A_{z1,2}\cos\theta + A_{r1,2}\sin\theta,$$

$$A_{\theta1,2} = -A_{z1,2}\sin\theta + A_{r1,2}\cos\theta,$$

$$A_{\varphi1,2} = A_{\varphi1,2}, \tag{11}$$

where θ is the azimuthal angle, the angle between the magnetic field and the direction of observation.

For the calculation of the integrals (10) the method of the stationary phase is applied. As a result:

$$A_z = A_{z1} + A_{z2} =$$
$$= \sum_{i=1}^{2}\sum_j \frac{e^{i\omega t}}{k_0 R}\left(\frac{d\chi_i}{d\Theta}\sin\Theta + \chi_i\cos\Theta\right)_{\Theta_j}\times$$
$$\times \mathrm{K}_i(\chi_i\sin\Theta_j) j_m(\chi_i\sin\Theta_j)\sqrt{\frac{\chi_i(\Theta_j)\sin\Theta_j k_0}{\sin\theta}}\times$$
$$\times\left(\frac{\exp\left(-ik_0RS_{+ij}(\Theta_j)-\frac{i\pi}{4}\left(1-\mathrm{sgn}\left|S_{+ij}^{//}(\Theta_j)\right|\right)\right)}{\sqrt{\left|S_{+ij}^{//}(\Theta_j)\right|}}+\right.$$
$$\left.+\frac{\exp\left(-ik_0RS_{-ij}(\Theta_j)+\frac{i\pi}{4}\left(1+\mathrm{sgn}\left|S_{-ij}^{//}(\Theta_j)\right|\right)\right)}{\sqrt{\left|S_{-ij}^{//}(\Theta_j)\right|}}\right)$$

$$A_r = A_{r1} + A_{r2} =$$
$$= \sum_{i=1}^{2}\sum_j \frac{e^{i\omega t}}{k_0 R}\left(\frac{d\chi_i}{d\Theta}\sin\Theta + \chi_i\cos\Theta\right)_{\Theta_j}\times$$
$$\times K_i(\chi_i\sin\Theta_j) j_m(\chi_i\sin\Theta_j)\sqrt{\frac{\chi_i(\Theta_j)\sin\Theta_j k_0}{\sin\theta}}\times$$
$$\times\frac{\chi_i^2\sin\Theta_j\cos\Theta_j\left(\chi_i^2-\varepsilon_\perp k_0^2\right)}{\left(\chi_i^2-\varepsilon_\perp k_0^2\right)\left(\chi_{//i}^2-\varepsilon_\perp k_0^2\right)-\alpha^2k_0^4}\times$$
$$\times\left(\frac{\exp\left(-ik_0RS_{+ij}(\Theta_j)+\frac{i\pi}{4}\left(3+\mathrm{sgn}\left|S_{+ij}^{//}(\Theta_j)\right|\right)\right)}{\sqrt{\left|S_{+ij}^{//}(\Theta_j)\right|}}-\right.$$
$$\left.-\frac{\exp\left(-ik_0RS_{-ij}(\Theta_j)-\frac{i\pi}{4}\left(3-\mathrm{sgn}\left|S_{-ij}^{//}(\Theta_j)\right|\right)\right)}{\sqrt{\left|S_{-ij}^{//}(\Theta_j)\right|}}\right)$$

$$A_\varphi = A_{\varphi1} + A_{\varphi2} =$$
$$= \sum_{i=1}^{2}\sum_j \frac{e^{i\omega t}}{k_0 R}\left(\frac{d\chi_i}{d\Theta}\sin\Theta + \chi_i\cos\Theta\right)_{\Theta_j}\times$$
$$\times K_i(\chi_i\sin\Theta_j) j_m(\chi_i\sin\Theta_j)\sqrt{\frac{\chi_i(\Theta_j)\sin\Theta_j k_0}{\sin\theta}}\times$$
$$\times\frac{\chi_i^2\sin\Theta_j\cos\Theta_j\alpha k_0^2}{\left(\chi_i^2-\varepsilon_\perp k_0^2\right)\left(\chi_{//i}^2-\varepsilon_\perp k_0^2\right)-\alpha^2k_0^4}\times$$
$$\times\left(\frac{\exp\left(-ik_0RS_{+ij}(\Theta_j)+(i\pi/4)\left(3+\mathrm{sgn}\left|S_{+ij}^{//}(\Theta_j)\right|\right)\right)}{\sqrt{\left|S_{+ij}^{//}(\Theta_j)\right|}}-\right. \tag{12}$$
$$\left.-\frac{\exp\left(-ik_0RS_{-ij}(\Theta_j)-(i\pi/4)\left(3-\mathrm{sgn}\left|S_{-ij}^{//}(\Theta_j)\right|\right)\right)}{\sqrt{\left|S_{-ij}^{//}(\Theta_j)\right|}}\right)$$

where $S_{\pm ij} = n_i(\Theta_j)\cos(\Theta_j - \theta)$, (13)

and where the values Θ_j correspond to the stationary phase points. In fact Θ is the propagation angle of the electromagnetic wave.

From (12) one can see that the radiation field is formed as an interference of several waves with different wave vectors.

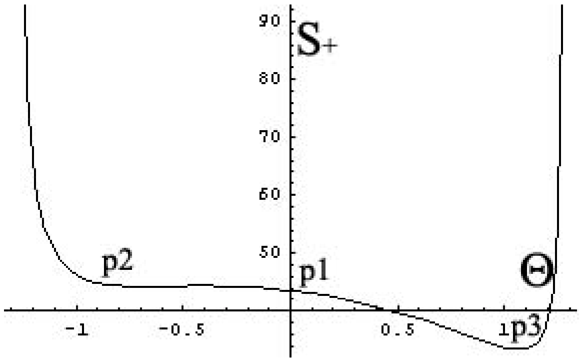
5. STATIONARY PHASE POINTS

To find out the stationary phase points it is necessary to solve the equation:

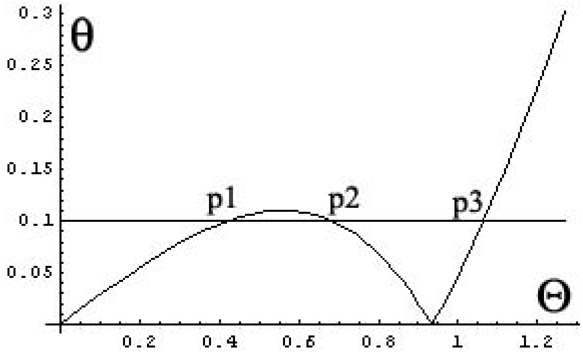
$$\frac{dS_{\pm ij}}{d\Theta} = 0 \qquad (14)$$

that can be presented in the form:

$$tg\theta = \pm\frac{f_{1,2} - tg\Theta}{1 + f_{1,2}tg\Theta}; \qquad f_{1,2} = \frac{1}{2n_{1,2}^2}\frac{dn_{1,2}^2}{d\Theta}, \qquad (15)$$



a



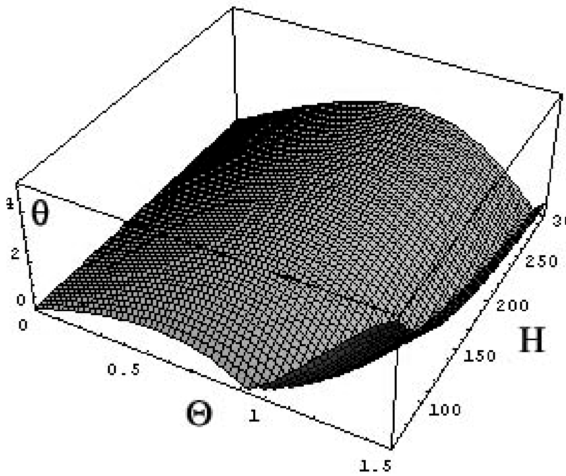
b

Fig. 1: a - S_+ versus the propagation angle of the electromagnetic wave for H=60G, n_p=1.4 $10^{11}cm^{-3}$, f_m=50MHz, b – azimuthal angle versus the propagation angle of the electromagnetic wave for the same parameters.-Stationary phase points are indicated

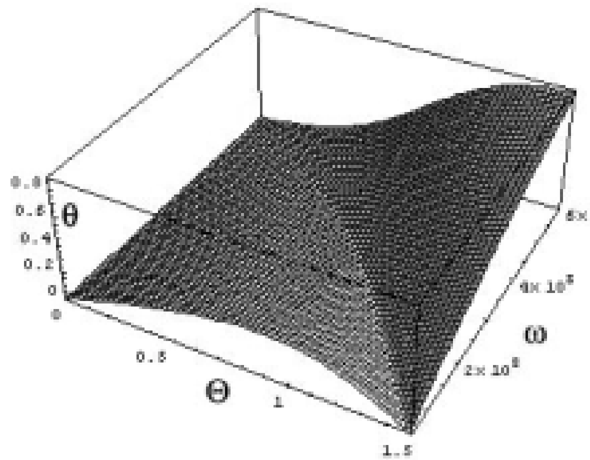
where $n_{1,2}$ are the roots of the dispersion relation (5). The equation (15) cannot be solved analytically and therefore numerical methods are used.

The dependence S(Θ) is shown on the Fig.1,*a*. The extrem *p1*, *p2* and *p3* correspond to the stationary phase points. The point *p1* has an analogue for electromagnetic waves in vacuum, the points *p2* and *p3* are specific for magnetoactive plasma. They appear due to the sharp increase of the wave number near the angle Θ corresponding to the resonance cone.

The values of the stationary phase points versus the azimuthal angle for the observation point have been calculated numerically. They are shown on the Fig.1,*b* that plots the angle of observation as a function of the propagation angle. Figs.2,*a-b* illustrate the influence of the magnetic field H and the modulation frequency on the stationary phase points' values.



a



b

Fig. 2. Number of stationary points versus the magnetic field H (in Gauss) for n_p = 1.2 $10^{11}cm^{-3}$, f_m=50 MHz (a) and versus the modulation frequency $\omega=2\pi f_m$ (in rad/s) for H=60G, n_p = 1.2 $10^{11}cm^{-3}$ (b)

A subsequent calculation shows that the point *p3* corresponding to the largest curvature gives the main contribution to the radio-emission. But this point does not exist for all possible values of the model parameters (see Fig.2). In particular it disappears when the usual conditions for whistler approximation

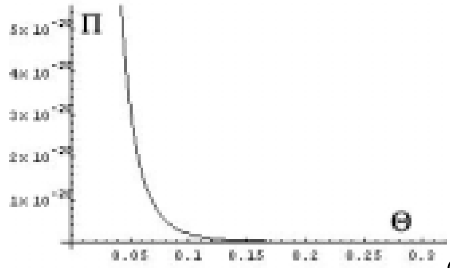
$$\omega << \omega_c << \omega_p. \qquad (16)$$

are satisfied.

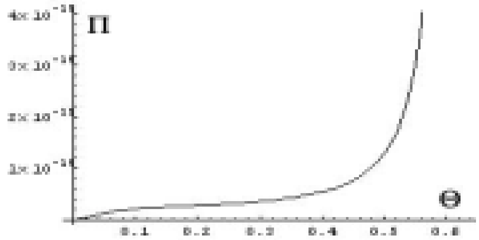
6. RADIATION PATTERN FOR DIFFERENT PARAMETERS OF THE MODEL

The radiation pattern (i.e. the angular dependence of the radial component of the Pointing vector) is shown on the Figs.3,*a-b*. Fig.3,*a* shows this dependence for the case when the conditions (16) are satisfied and the point *p3* disappears. The shape of the radiation pattern for this case conforms to the results obtained in [4].

Fig.3,*b* is plotted for parameters corresponding to the experimental conditions [2]. For this case the point *p3* gives the main contribution to radioemission.



a



b

Fig.3 Angular dependence of the radial component of the Pointing vector (in arbitrary units): a – H=300G, n_p =3.5 $10^{12}cm^{-3}$, f_m=50MHz; b – H=60G, n_p =1.2 $10^{11}cm^{-3}$, f_m=100MHz

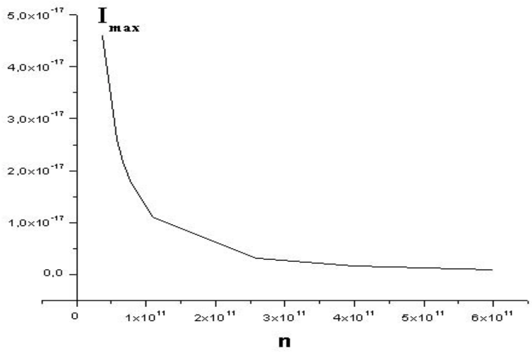


Fig. 4. Maximum energy flow density (in arbitrary units) versus the plasma density (in cm^{-3}) for H=60G, f_m=100MHz

The dependence of the maximum energy flow density versus the plasma density for that case is shown on the Fig.4. One can see that the increase of the plasma density results in the decrease of the transition radiation intensity.

The Fig.5 [2], shows the variation of the intensity of the transition radiation harmonics as a function of time, that is, as a function of the decreasing plasma density (the appearance of upper harmonics is caused by the anharmonicity of the modulation beam current). One can see that the intensity of the harmonics radiation (particularly for the second and the third harmonics) increases (to some degree). These results qualitatively conform to our calculations (Fig.4).

7. CONCLUSION

The transition radiation of a thin modulated electron beam injected from a conducting plane into a plasma along an arbitrary magnetic field normal to that plane is calculated. The radiation field is formed as a result of the interference of three waves with different wave vectors. The radiation pattern is mainly determined by one of those waves (it depends on the parameters of the model).

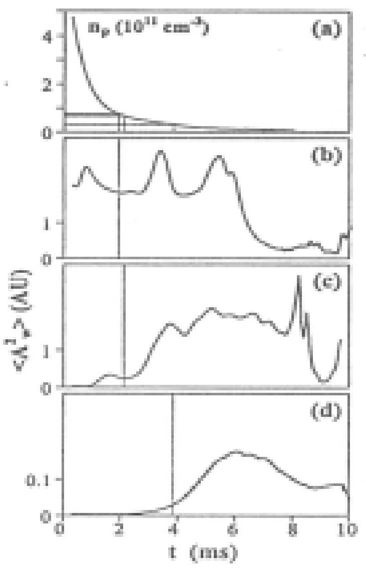


Fig. 5. a –Variation of the plasma density n_p (in cm^{-3}) as a function of the time; amplitudes of the whistler waves (in arbitrary units) as function of time for H=60G, f_m=50MHz (b), 2 f_m =100MHz (c), 3 f_m=150MHz (d)

For the case of the whistler approximation, the obtained results coincide with our previous calculations [4]. The calculated dependence of the radiation intensity versus the plasma density qualitatively agrees with the experimental data [2].

In order to perform a precise comparison of the calculation results and the experimental data it is necessary to take into account the finite radius of the electron beam, to calculate the radiation in the near-field region and to examine the case of the beam injection at some angle to the magnetic field (this case corresponds to the conditions of the experiment [5]).

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