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ION ACCELERATION IN SCANNING WAKE-FIED ACCELERATOR

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We propose a concept of high-gradient acceleration in a Scanning Wake-Field Accelerator (SWFA). Advantages inherent to this concept make it possible to create a compact accelerator with a stable, long-duration and controlled acceleration in a wide range of velocities, that is very significant for ion acceleration with a small ratio Z_i/M_i (Z_i is the ionization state, M_i – ion mass).

Further progress in the technology of a linear particle accelerator depends on the development of radically new acceleration methods, making it possible to avoid a breakdown on material surfaces and achieve ultrahigh accelerating gradients. In this context, plasma-based methods [1-4], using for acceleration the longitudinal plasma waves excited in plasma by an electron beams (EB) or laser beams (LB) are very attractive ones. In fact, it has been experimentally confirmed [5-9] the reality of creating super-high gradients by these methods, but such processes as a LB diffraction, instabilities, energy depletion of the drivers (EB or LB), phase detuning between the wave and accelerated particles strictly limit the acceleration length and hence the energy gain. To overcome these difficulties, we propose a concept, which we call the Scanning Wake-Field Accelerator (SWFA) that combine the virtues of the linear scanned drivers [10,11] and the Laser Wake-Field Accelerator (LWFA) [3] or the Plasma Wake-Field Accelerator (PWFA) mechanism [2]. By the linear scanning, the photons or electrons of the drivers are injected into the narrow plasma channel transversely to its axis, unlike the others novel methods investigated so far, and simultaneously the window of the light (electrons) emission is moved parallel to the axis under control. This lead to the controlled motion along the channel the LB′s ponderomotive force (EB's coulomb force) that exert on plasma electrons and, under some conditions, excite behind itself the wake fields which can be used for acceleration.

The proposed accelerating scheme which use the LB as a scanned driver has following advantages: i) The limitation on the acceleration distance, caused by the diffraction is avoided; ii) a laser-plasma stability is essentially improved due to the greatly reduced time of photons presence in the near-axis region compared to the acceleration time; iii) driver′s energy along the acceleration axis is keeping constant, because in the every next region of the channel the new and fresh portions of photons will appear; iiii) at last, due to aforementioned advantages and scanning, the controlled acceleration can be realized in a wide range of accelerating velocities that permit to accelerate ions with a small initial velocity and small ratio Z_i/M_i.

LINEAR SWFA MODEL

We use the 2-D equation, as a basic one, describing the potential longitudinal plasma waves (wake waves) driven in the plasma by ponderomotive forces of the cylindrical LB being space-limited in the axial (z) and radial (r) directions.

$$\left(\frac{\partial^2}{\partial t^2}+\omega_P^2\right)\Phi(r,z,t)=\omega_P^2\Phi_L(r,z,t), \qquad (1)$$

This equation has been developed [3] for cold plasma in a linear, weak-relativistic approach $a=\mathrm{v}_{osc}/c<1$, (where v_{osc} is the jitter of the electron in the laser radiation field) and $\omega_L>>\omega_P$, (here ω_L is the laser frequency, $\omega_P=\sqrt{\frac{4\pi ne^2}{m_e}}$ and n is an ambient plasma density). Equation (1) represents a simple harmonic oscillator for the wake plasma potential Φ driven by the ponderomotive potential (PP) $\Phi_L=\frac{m_ec^2}{2e}\langle\bar{a}^2(r,z,t)\rangle$, where $\bar{a}(r,z,t)=\frac{e\bar{A}(r,z,t)}{m_ec^2}$ is the normalized vector potential of the laser field. For the linear polarized light, after averaging over $2\pi/\omega_L$, PP takes the form $\Phi_L=\left(m_ec^2|a_0|^2/4e\right)f(r,z,t)=\varphi_L\cdot f(r,z,t)$ were $|a_0|^2\cong 7.3\cdot10^{-19}\lambda_L^2[\mu m]I$ [W·cm^{-2}], λ_L - laser wave length, I is the LB′s intensity and $f(r,z,t)$ is the function describing the LB′s intensity envelope. Usually, in the linear and quasi-stationary approach, for the bi-Gaussian beam, the PP can be written in the independent variables (ξ,τ) of the frame, moving with a LB group velocity $\mathrm{v}_g\approx c$ as

$$\Phi_L=\varphi_L e^{-r^2/\sigma_r^2}\cdot e^{-\xi^2/\sigma_Z^2}, \qquad (2)$$

where $\xi=z-\mathrm{v}_g t$ and σ_r,σ_z are the widths of LB at $1/e$ in intensity, in a radial and axial direction, respectively.

In our model we put $e^{-\xi^2/\sigma_Z^2}=1$ in (2) and use the transverse PP, that is valid for a long $(\sigma_z\to\infty)$ LB or for a region in the vicinity $(\xi<<\sigma_z)$ of LB focus.

The geometry of the model is represented in Fig.1 in Cartesian system. The LB cross-section shown in Fig.1 has the elongation in the x direction. Scan is realized in the y direction with a linear acceleration a_i and velocity $v_y = v_0 + a_i t_s$, assumed to be less than the light one, where v_0 is the initial velocity, t_s - the time in the laboratory frame.

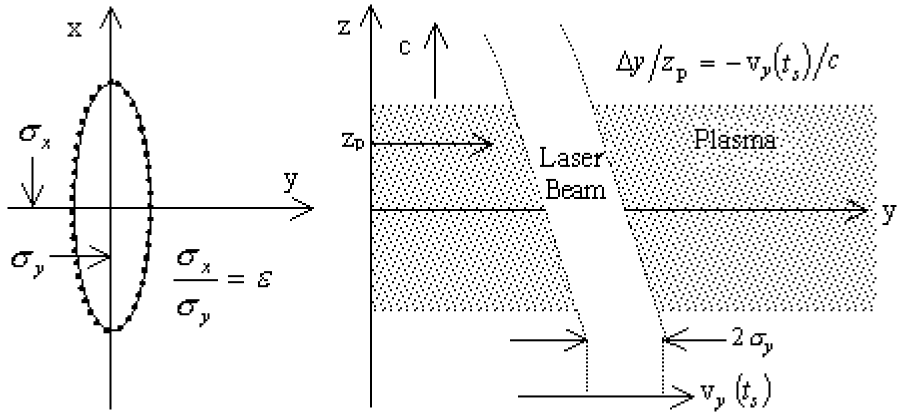

Fig.1. The geometry of the SWFA model

Transverse PP in this geometry and in the variables (ξ, τ) of the frame moving with a scanned velocity v_y is written as

$$\Phi_L = \varphi_L e^{-x^2/\sigma_x^2} \cdot e^{-\xi^2/\sigma_y^2}, \quad (3)$$

where σ_x, σ_y are the LB half-widths at $1/e$ in intensity in the x, y directions, $\sigma_x > \sigma_y$, $\varepsilon = \sigma_x/\sigma_y$ is a parameter of the elongation (See Fig.1) and

$$\xi = -y + v_0(t_s - \tau) + \frac{a_i(t_s - \tau)^2}{2} - \beta_s(\tau) z_P,$$

where $\beta_s(\tau) = \frac{v_0 + a_i(t_s - \tau)}{c}$.

Appearance of the fourth term in ξ is typical for the linear scanned beams and obvious from the geometric consideration (See Fig.1). Note, that ξ depends parametrically on time t_s and coordinates (y, z_P) of the laboratory frame. In the new variables τ changes from 0 to t_s, that corresponds to ξ increasing in the negative direction $(\xi < 0)$. z_P is very small and for the intensive LB is restricted to $|z_P| \le z_R$, where $z_R = k_L \sigma_y^2$ is the Raileigh diffraction length and k_L - the laser radiation wave number. So, in the new variables with a PP given by (3), taking into account quasistatic approximation $(\partial/\partial\tau = 0)$, Eq.(1) become

$$\left(\frac{\partial^2}{\partial\xi^2} + \frac{a_i}{v_\xi^2(\tau)} \frac{\partial}{\partial\xi} + \frac{\omega_P^2}{v_\xi^2(\tau)} \right) \times \Phi(x,\xi) = \frac{\omega_P^2}{v_\xi^2(\tau)} \Phi_L(x,\xi) \quad (4)$$

where $v_\xi(\tau) = v_0 + a_i(t_s - \tau)$.

This equation can be simplified. At first, we assume that $\omega_P^2 / v_\xi^2(\tau) = k_P^2 = const$. This means that the acceleration must be realized in the plasma with density linearly increasing along the $axis(y)$, $n = n_0 + D \cdot y$, where D is a density gradient. In order that the quasistatic state for the density be valid the relative density change on the LB width $(2\sigma_y)$ must be neglected. In practice this condition is satisfied $\frac{\Delta n}{n} \approx \frac{n_f}{n_0} \frac{2\sigma_y}{L_a} << 1$, where n_0, n_f are the initial and final densities on the acceleration length L_a. Then, we neglect the second term on the left-hand side of Eq.(4) provided that the accelerating field $E_a << \frac{4W_0(eV)}{Z_i \sigma_y (cm)}$.

At the initial ion energy $W_0 \approx 1$ MeV, $\sigma_y \approx 10^{-3} cm$ and $Z_i = 1$ we have $E_a << 400$ GV/m. At last we obtain the equation being formally the same one as usually used for describing the wake plasma wave following the short intensive laser pulse

$$\left(\frac{\partial^2}{\partial\xi^2} + k_P^2 \right) \Phi(x,\xi) = k_P^2 \Phi_L(x,\xi).$$

In the wake approach $(\xi << -\sigma_y)$ the solution is

$$\Phi = k_P^{-1} E_A \cdot f(x) \cdot R(k_P \sigma_y) \times \times \sin k_P \xi = k_P^{-1} A(x, k_P \sigma_y) \sin k_P \xi, \quad (5)$$

where $E_A = \frac{4\sqrt{\pi}}{e_{xp}} \frac{\varphi_L}{\sigma_y}$ is the wake field amplitude,

$$f(x) = e^{-x^2/\sigma_x^2} \quad \text{and}$$

$$R(k_P\sigma_y) = \frac{k_P^2\sigma_y^2}{4}\exp\left[1 - \frac{k_P^2\sigma_y^2}{4}\right]$$

is the quasiresonant multiplier equal to unity at $k_P\sigma_y = 2$ (Fig.2). The accelerating field defined from (5) is $E_y = A(x, k_P\sigma_y)\cdot\cos k_P\xi$. The field amplitude, normalized by its maximum value $E_{y\max} = A(x = 0, k_P\sigma_y = 2)$ equal to $E_y^* = R(k_P\sigma_y)$.

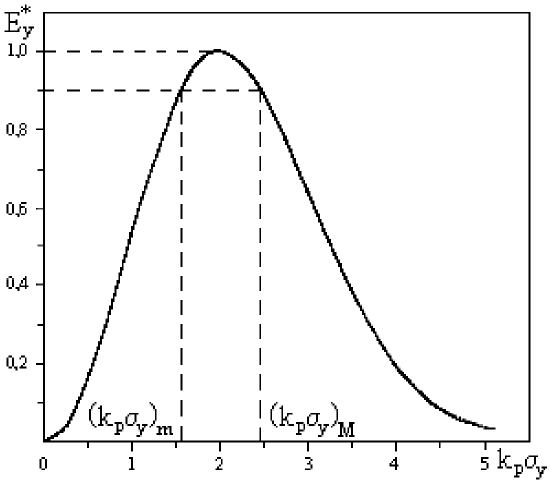

Fig.2. Normalized amplitude of the accelerating wave field $E_y^* = R(k_P\sigma_y)$. At the fixed v_ξ and σ_y this curve reflects E_y^* dependence on the plasma electron density $(k_P\sigma_y \approx \sqrt{n_e})$ change in the fixed cross-section of the accelerating channel

The quasiresonant curve $R(k_P\sigma_y)$ shown in Fig.2 demonstrates the wake field reaction on the density change in the fixed plasma channel cross-section (fixed σ_y and v_ξ). It follows from Fig.2 that the normalized wave field amplitude E_y^* remains in the range ($0.9 \le R \le 1$) when the density is changed by a factor of 2.5 $\left(\frac{(k_P\sigma_y)_M^2}{(k_P\sigma_y)_m^2} = \frac{n_M}{n_m} \approx 2.5\right)$. So, the proposed concept does not require the fine tuning of the plasma density profile along the acceleration channel. The x and z field components are

$$E_x = -\frac{2x}{\sigma_x^2}\cdot\Phi \qquad E_z = \beta_s(\tau)E_y .$$

E_z does not change the sign with the sign of the z_P and thus this field is defocusing.

One of the ways for compensation of E_z and providing the focusing in this direction, consist in the simultaneous scanning of two injected opposed LB due to which the resulting z-component of the wake field acquires the focusing quality.

$$E_z^\Sigma = -2\beta_s(\tau)\times$$
$$\times A(x, k_P\sigma_y)\sin(k_P\xi_0)\sin(k_P\cdot\beta_s(\tau)\cdot z_P),$$

where $\xi_0 = \xi(z_P = 0)$.

The same focusing effect in average can be obtained by changing periodically the direction of the LB injection on the reverse along the axis.

We illustrate potentialities of the SWFA method by calculating the acceleration of an ion with $Z_i/M_i=10^{-2}$. It is known that the wave fields values are limited by the so called "wave breaking" amplitude E_{WB}, proportional to the square of phase velocity. This proportion is shown in Fig.3, where used was the expression for E_{WB} taken of [12] which was obtained for the warm plasma and nonrelativistic wave

$$E_{WB} = \frac{m_e v_{ph}^2}{e}k_P F(\beta), \tag{6}$$

here v_{ph} is the phase (ion) velocity, $F(\beta) = \left(1 - \frac{1}{3}\beta - \frac{8}{3}\beta^{1/4} + 2\beta^{1/2}\right)^{1/2}$, $\beta = \frac{3T}{m_e v_{ph}^2}$, T is the electron thermal energy, which was set equal 10 eV in our calculations. In the same Fig.3 the plasma density is plotted as a function of the velocity in accordance with the expression $\omega_p^2 = k_P^2 v_{ph}^2$ and $k_P = 10^4$. It follows that the velocity $v_{ph} = 8\,10^8$ cm/s corresponds to $E_{WB} \approx 130$ MV/m. Estimations show that at these gradient, velocity and $T \le 10$ eV, we can neglect of the collisional and Landau damping.

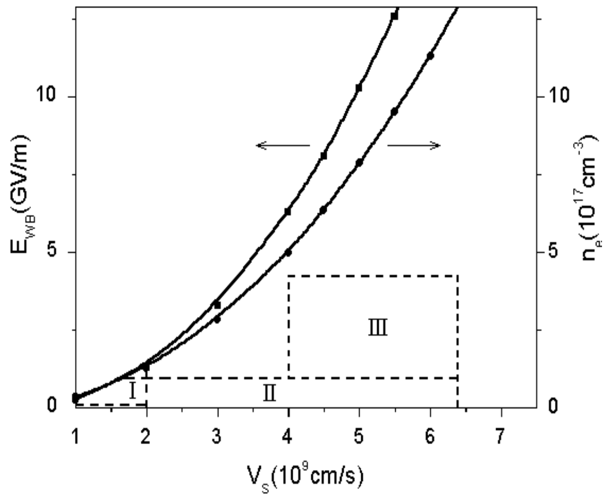

Fig.3. The wave-breaking electric field amplitude

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E_{WB} and the plasma electron density n_e as a function of the scanned wake wave velocity v_ξ ($v_\xi = v_{ph} << c$) from (6) [12] and expression $\omega_p^2 = k_P^2 v_{ph}^2$. The parameters are T_e =10eV and $k_P = 10^4$. Dashed line indicates the stages of the acceleration

Taking into account the E_{WB} dependence on the velocity, calculations were carried out for two and three acceleration stages (this is marked in Fig.3). At the first stage the gradient was taken equal to E_a=100 MV/m. At the following stages the gradients were increasing but remaining less than E_{WB}. Of course, the acceleration could be performed continuously in a single stage that results in shortening the acceleration length, but this requires the nonlinear density and velocity dependence on the distance and the time and increasing with a distance of the LB′s intensity. In calculations the following input parameters were used: $Z_i/M_i=10^{-2}$, laser wavelength $\lambda_L = 1\mu m$, $\sigma_y = 2\lambda_L$, $\varepsilon = \sigma_x/\sigma_y = 3$, gradients E_a=(0.1;1;4)GV/m, initial ion energy W_0=33 MeV, final energy W_f=2.1 GeV. The calculation results are listed in Table: We see that in the course of the ion acceleration from initial energy to the final one the density is increasing along the axis from n_0=2$10^{16}$cm^{-3} to n_f≈1.3$10^{18}$cm^{-3}.

E_{ya} GV/m	W_0 (GeV)	W_f (GeV)	n_0 10^{17}cm^{-3}	n_f 10^{17}cm^{-3}	L_a (m)	τ_a 10^{-9}s	Q_L 100J	P_L GW	I_L 10^{16} W/cm^2	$\frac{\delta\tilde{n}}{n_0}$	$\frac{\delta\tilde{n}}{n_f}$	a
0.1	0.03	0.21	0.2	1.3	1.8	120	0.46	0.4	0.1	0.3	0.04	0.03
1	0.21	2.1	1.3	13	1.9	45	1.7	3.8	1	0.5	0.05	0.1
1	0.21	0.81	1.3	5	0.6	20	0.76	3.8	1	0.5	0.13	0.1
4	0.81	2.1	5	13	0.3	6	0.91	15	4	0.5	0.2	0.2

The total length of the three-stage accelerator is L_a^Σ≈2.7 m, the LB energy Q_L^Σ≈200 J, LB duration $\tau_L^\Sigma \cong$ 140 nsec. One can see that LB energetic parameters are in the frame of the present-day technology. In the Table also listed are the values of the LB power P [W], intensity I [W/cm^2], the wave linearity parameter $\frac{\delta\tilde{n}}{n}$ and the relativistic one $a = eE_L/m_e\omega_L c$.

CONCLUSION

Simple analysis and estimations of the SWFA concept show a possible way to realize the stable, controlled and long-term acceleration with the superhigh gradients. As an example, the ion acceleration with $Z_i/M_i=10^{-2}$ was calculated and it was shown that even at the initial ion velocity ≈8.10^{8}cm/s one can obtain the accelerating gradients significantly exceeding those achievable at the same velocity in conventional accelerators. Estimated LB energetic parameters required for acceleration of heavy ions up to energy of 2.1 GeV are quite feasible for the now-day technology. Nevertheless, before the next steps of research in this direction we must elucidate some questions related to realization of the linear scanning and the optimum LB shape as well as to the possibility of the LB multiple round-trip usages in the accelerator to raise the LB energy efficiency. It will be also useful to compare the accelerators, which use either the LB or the EB as a scanned driver.

REFERENCES

1. Ya.B. Fainberg // *Proceeding of the CERN Symposium on High Energy Accelerators and Pion Physics.* CERN, Geneva, 1956, p.84.
2. P. Chen, J.M.Dawson, R.W. Huff and T. Katsouleas // *Phys. Rev. Lett*. 1985, 54, 693.
3. L.M. Gorbunov, V.I. Kirsanov // *Zh. Eksp. Teor. Fiz.* 1987, 93, 509, // *Sov. Phys. JETP*, 1987, 66, 209.
4. Sprange et. al. // *Appl. Phys. Lett.* 1988, 53, 2146.
5. K. Nakajiama // *Phys. Plasmas*. 3(5), 1996, 2169.
6. H. Dewa et. al. // *Joint ICFA/JAERI-Kansai Int.* Workshop, Kyoto, Japan, July 14-18, 1997.
7. A. Modena et.al. Nature (London), 1995, 377, 606.
8. C.W. Siders et.al. // *Phys. Rev. Lett.,* 1996, 76, 3570.
9. J.R. Marques et.al. // *Phys. Rev. Lett.,* 1996, 76, 3566, 1997, 78, 3463.
10. V.H. Alfven and O. Wernholm // *Arkiv for Fysik.* 1952, 5(12), 175.
11. А. с. СССР, МКИ^3 Е.М. Лацько, Открытия. Изобретения. 1989, №1529476.
12. T.P. Coffey // *Phys. Fluids*, 1971, 14, 1402.

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